SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find the standard form of the equation of the ellipse satisfying the given conditions.

1) Foci: (0, –2), (0, 2); y–intercepts: –5 and 5

Note: graphs of conic sections for problems 1 to 12 were made with the Algebra (Main) App, available at www.mathguy.us/PCApps.php.

This ellipse has foci $(0, \pm 2)$, and therefore has a vertical major axis.

The standard form for an ellipse with a vertical major axis is:

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

The values of a and b can be determined from the foci and the y-intercepts.

The foci are located at $(h, k \pm c) = (0, \pm 2)$, where $c^2 = a^2 - b^2$. So, we can determine that:

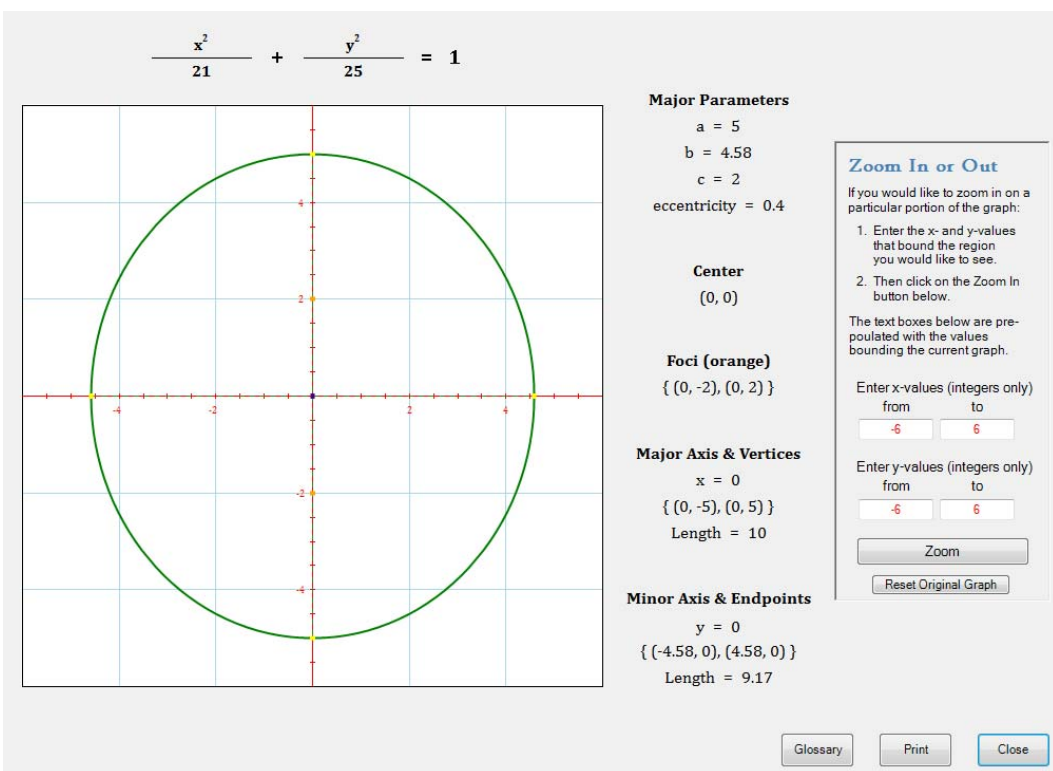
- $h = 0,\ k = 0,\ c = 2$

The y-intercepts, then, are major axis vertices, which are located at $(h, k \pm a) = (0, \pm 5)$. So, we determine that:

- $a = 5$
- $c^2 = a^2 - b^2$ give us $2^2 = 5^2 - b^2$, so $b^2 = 21$

Then, substituting values into the standard form equation gives:

$$\frac{x^2}{21} + \frac{y^2}{25} = 1$$



Find the standard form of the equation of the hyperbola satisfying the given conditions.

2) Foci: (0, –4), (0, 4); vertices: (0, –3), (0, 3)

This hyperbola has foci $(0, \pm 4)$, and therefore has a vertical transverse axis.

The standard form for a hyperbola with a vertical transverse axis is:

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

The values of a and b can be determined from the foci and the vertices.

The foci are located at $(h, k \pm c) = (0, \pm 4)$, where $c^2 = a^2 + b^2$. So, we can determine that:

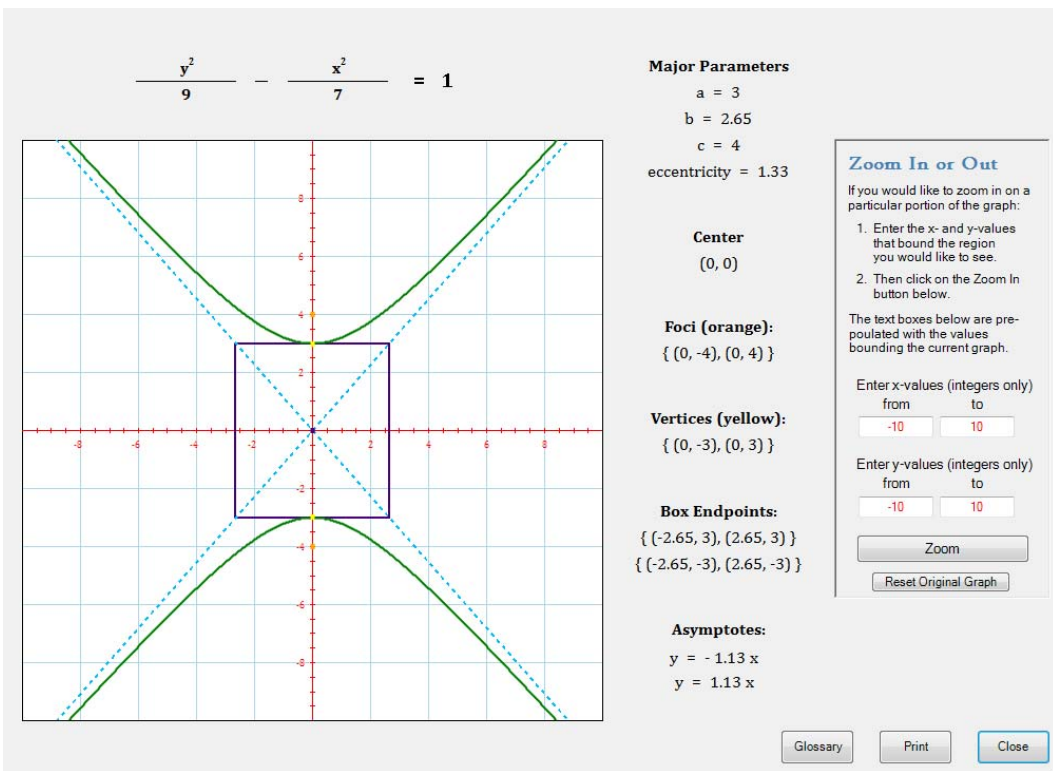
- $h = 0,\ k = 0,\ c = 4$

The vertices are located at $(h, k \pm a) = (0, \pm 3)$. So, we determine that:

- $a = 3$
- $c^2 = a^2 + b^2$ gives us $4^2 = 3^2 + b^2$, so $b^2 = 7$

Then, substituting values into the standard form equation gives:

$$\frac{y^2}{9} - \frac{x^2}{7} = 1$$



Find the standard form of the equation of the parabola using the information given.

3) Vertex: (2, –5); Focus: (2, –7)

This vertex and focus of this parabola have the same x-coordinate. Therefore the parabola has a horizontal Directrix. The focus is below the vertex on a graph, so the parabola opens down.

The standard form for a parabola with a horizontal Directrix, according to Blitzer, is:

$$(x-h)^2 = 4p(y-k)$$

Note: There are varying opinions as to what the standard form of a parabola is. The student should use whatever form is preferred by their teacher.

The value of p can be determined from the focus and the vertex.

The vertex is located at $(h,k) = (2,-5)$. So, we determine that:

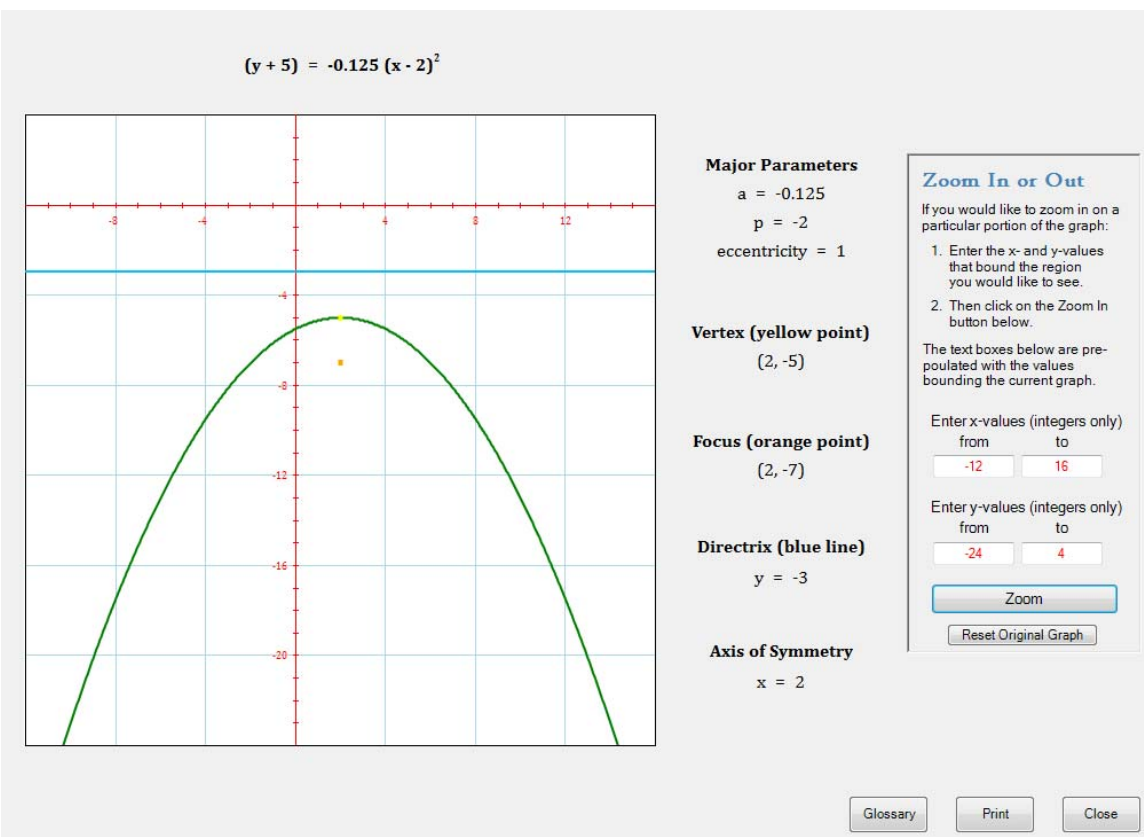
- $h = 2,\ k = -5$

The focus are located at $(h, k+p) = (2,-7)$. So, we can determine that:

- $p = (k+p) - k = -7 - (-5) = -2$

Then, substituting values into the standard form equation gives:

$$\mathbf{(x-2)^2 = -8(y+5)}$$



Identify the equation without completing the square.

4) $y^2 - 2x^2 + 3x + 4y + 1 = 0$

This problem can be answered by considering only the square terms. So, consider $Ax^2 + By^2$.

Here are the rules for determining the type of curve when given a general conic equation:

If either A or B is missing (i.e., there is only one square term), the equation is a parabola.

If A and B both exist:

- If A and B have different signs, the equation is a hyperbola.
- If A and B have the same sign and the same coefficients, the equation is a circle.
- If A and B have the same sign and different coefficients, the equation is an ellipse.

This equation is a hyperbola because both A and B exist, and their signs are different.

Note, you may be interested in the flowchart for conic section classification included in the PreCalculus Chapter 9 Companion on www.mathguy.us.

Find a set of parametric equations for the rectangular equation.

5) $y = \frac{(x+2)^2}{2}$

There are an infinite set of parametric equations you can create to represent the equation given. To find one that is reasonable and useful, follow these steps:

a. Set t equal to some function of x that simplifies the expression.
b. Solve the equation in Step a for x in terms of t, to get the function $x(t)$.
c. Substitute the value of x in terms of t (i.e., from $x(t)$) into the equation and see what results for y in terms of t (i.e., $y(t)$).
d. If you like both $x(t)$ and $y(t)$, keep them. If not, repeat these steps until you have $x(t)$ and $y(t)$ that you like.

For this problem, a couple of set of parametric equations present themselves as fairly obvious. Both of these are acceptable solutions to this problem:

- If we let $t = x$, then $x = t$ and $y = \frac{(t+2)^2}{2}$, however this is very simplistic and boring.

  Solution 1: $\boldsymbol{x = t,\ y = \frac{(t+2)^2}{2}}$

- If we let $t = x + 2$, then $x = t - 2$ and $y = \frac{t^2}{2}$. This is more interesting.

  Solution 2: $\boldsymbol{x = t - 2,\ y = \frac{t^2}{2}}$

Graph the ellipse and locate the foci.

6) $9x^2 + 4y^2 = 36$

Divide by 36 to get this equation in standard form:

$\frac{x^2}{4} + \frac{y^2}{9} = 1$ Standard form is: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ or $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

Since the y-term has the larger denominator, we know that the ellipse has a vertical major axis.

The larger of the two denominators is a^2. Therefore:

- $a^2 = 9$, so $a = 3$
- $b^2 = 4$, so $b = 2$
- Also, note that: $h = 0$, $k = 0$

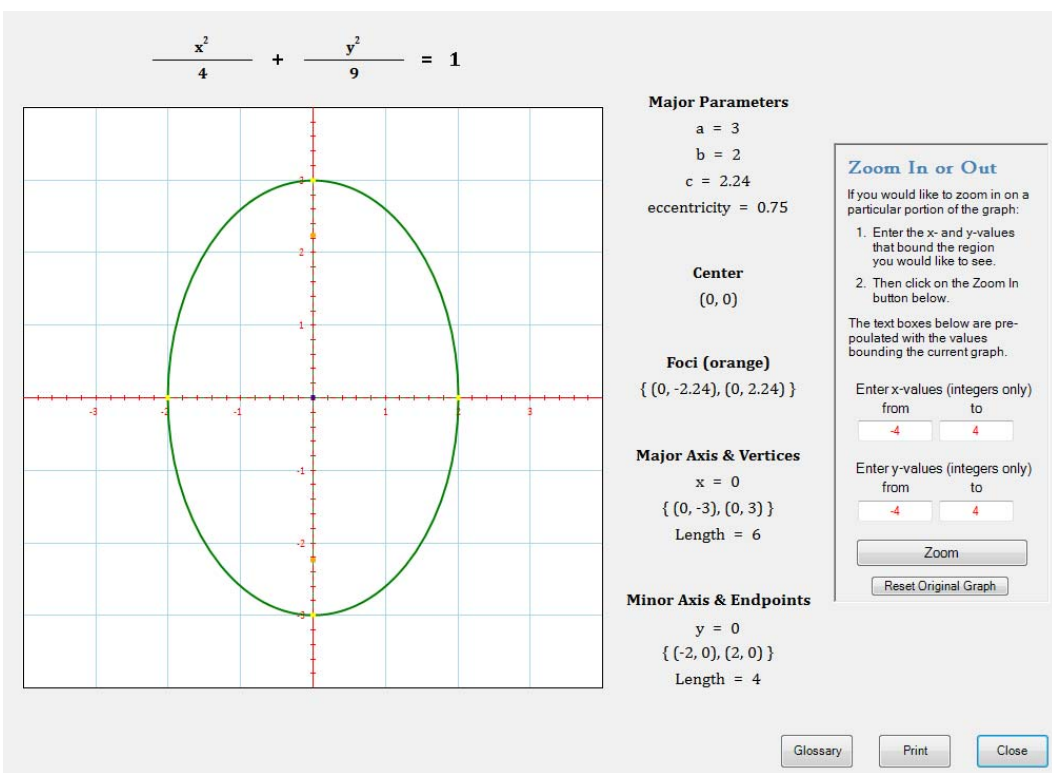
The foci for an ellipse with a vertical major axis are located at $(h, k \pm c)$, where $c^2 = a^2 - b^2$. So, we can determine that:

- $c^2 = 9 - 4 = 5$, so $c = \sqrt{5}$

The foci are located at $(h, k \pm c) = (0, \pm\sqrt{5})$

To graph the ellipse it is useful to first identify and graph the major and minor axis vertices.

- Major axis vertices exist at $(h,\ k \pm a) = (0, \pm 3)$
- Minor axis vertices exist at $(h \pm b,\ k) = (\pm 2, 0)$



Use the center, vertices, and asymptotes to graph the hyperbola.

7) $(y-4)^2 - 9(x+2)^2 = 9$

Divide by 9 to get this equation in standard form:

$\frac{(y-4)^2}{9} - \frac{(x+2)^2}{1} = 1$ Standard form is: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Since the y-term is positive, we know that the hyperbola has a vertical transverse axis.

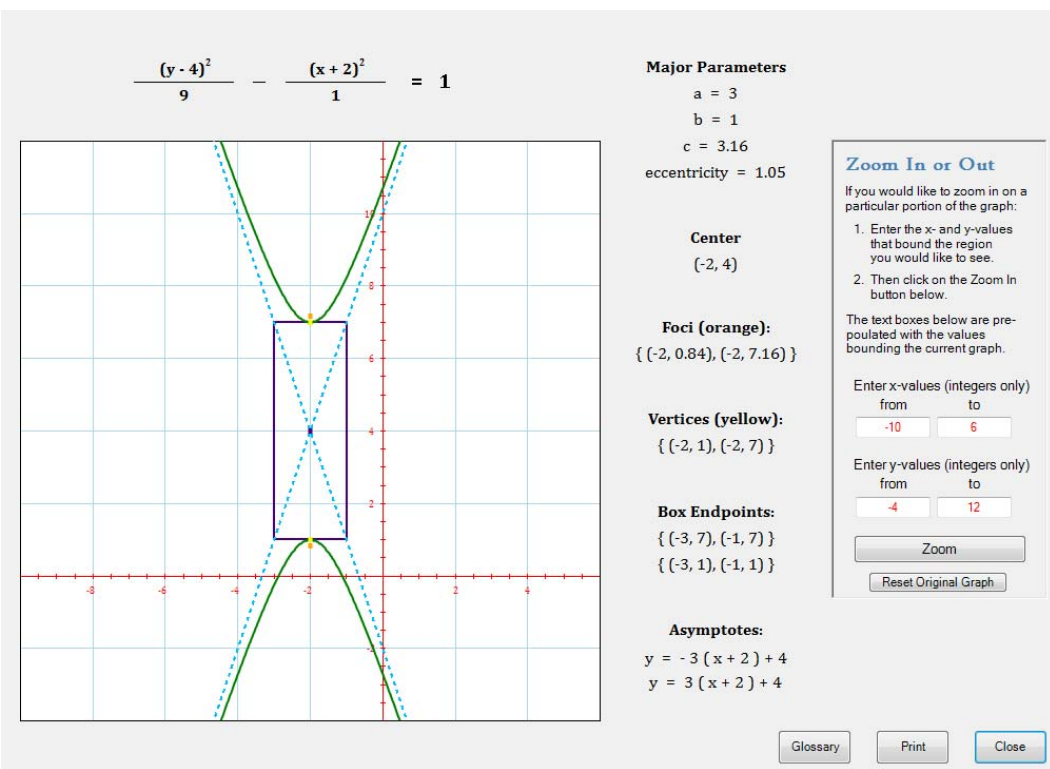
The vertices for a hyperbola with a vertical transverse axis are located at $(h, k \pm a)$. So, let's see what we can determine for this hyperbola:

- $h = -2,\ k = 4$
- $a^2 = 9$, so $a = 3$
- $b^2 = 1$, so $b = 1$
- **The vertices are:** $(-2, 4 \pm 3) = \{(-2, 1), (-2, 7)\}$
- **The center is halfway between the vertices:** $(-2, 4)$

The asymptotes for a hyperbola with a vertical transverse axis are $y - k = \pm\frac{a}{b}(x-h)$. So:

- **The asymptotes are:** $y - 4 = \pm 3(x+2)$

To graph the hyperbola, first graph the asymptotes and the vertices, and then sketch in the rest.



Graph the parabola with the given equation.

8) $(x + 2)^2 = -6(y - 1)$

Graphing parabolas is relatively easy compared to graphing ellipses or hyperbolas. The key to graphing a parabola is to identify its vertex and orientation (which way it opens). Consider the form of the above equation:

$$(x - h)^2 = 4p(y - k)$$

From this equation, we can determine the following:

- The vertex of the parabola is $(h, k) = (-2, 1)$.
- Since the x-tem is squared, the parabola has a horizontal Directrix (i.e., it opens up or down).
- Since $4p = -6$, we conclude that p is negative, so the parabola opens down.

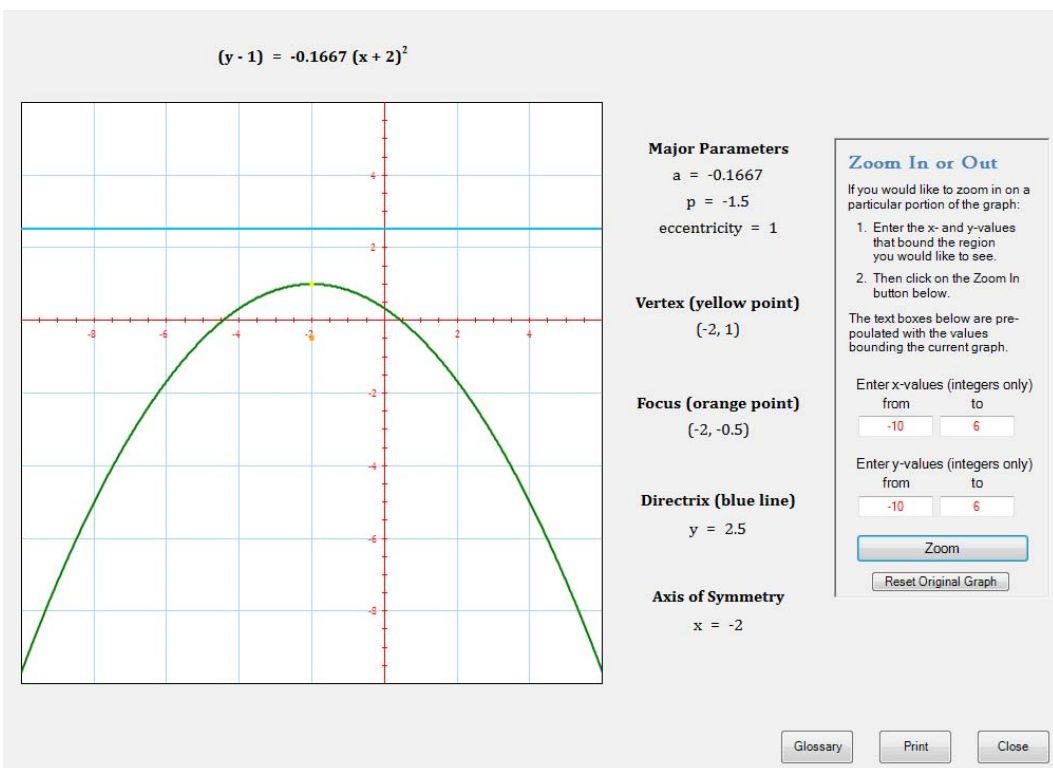
Let's find a couple of points to help us refine our graph of the parabola. Rewrite the equation in a simpler form to find y, given x.

$$y = -\frac{1}{6}(x + 2)^2 + 1$$

We already have a point – the vertex, at $(-2, 1)$. Let's find a couple more:

- Let $x = 4$. Then $y = -\frac{1}{6}(4 + 2)^2 + 1$. This gives us the point $(4, -5)$.
- Let $x = -8$. Then $y = -\frac{1}{6}(-8 + 2)^2 + 1$. This gives us the point $(-8, -5)$.

Now, graph the parabola.



Find the standard form of the equation of the ellipse satisfying the given conditions.

9) Endpoints of major axis: (–5, 4) and (3, 4); endpoints of minor axis: (–1, 1) and (–1, 7)

Since the major axis endpoints have the same y-value, the ellipse has a horizontal major axis.

An ellipse with a horizontal major axis has the following characteristics:

- The center is at (h, k), which is the midpoint of the major axis vertices, so:

$$(h,k) = \left(\frac{-5+3}{2}, 4\right) = (-1,4)$$

- Major axis vertices exist at $(h \pm a,\ k) = (-1 \pm 4,\ 4)$
- Minor axis vertices exist at $(h,\ k \pm b) = (-1,\ 4 \pm 3)$

From this we can determine that:

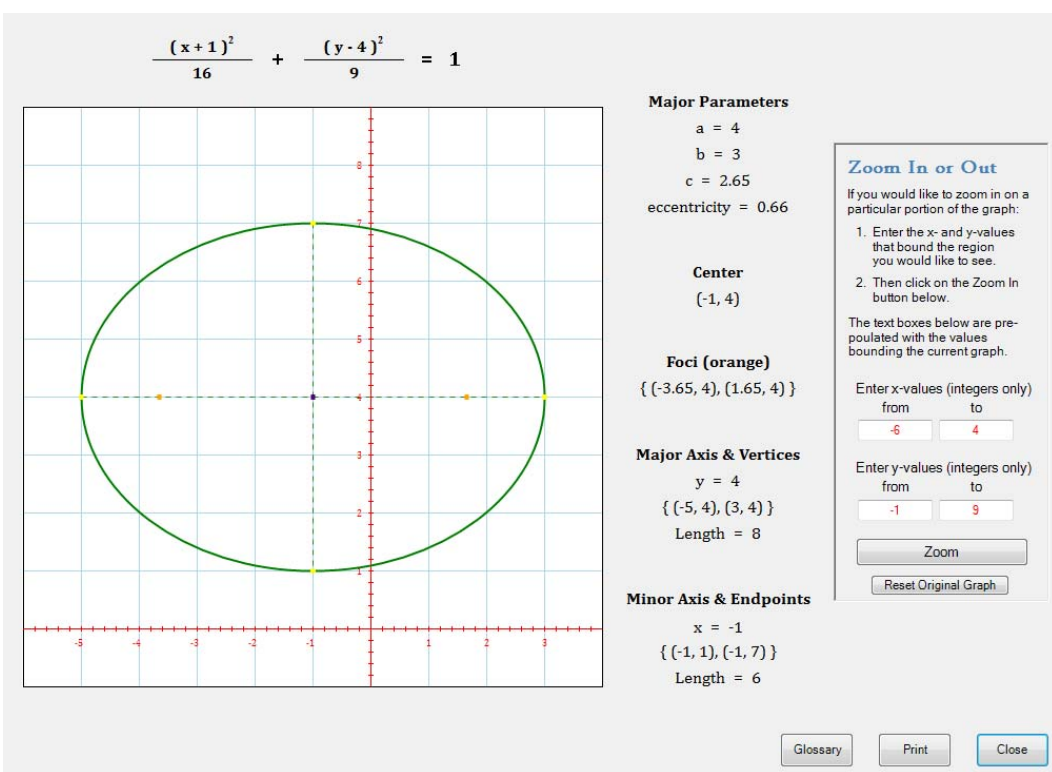
- $a = 4$
- $b = 3$

Standard form for an ellipse with a horizontal major axis is: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

So the standard form for the ellipse defined above is:

$$\frac{(x+1)^2}{16} + \frac{(y-4)^2}{9} = 1$$

We are not required to graph the ellipse, but here's what it looks like:



Find the standard form of the equation of the hyperbola satisfying the given conditions.

10) Endpoints of transverse axis: (0, −10), (0, 10); asymptote: $y = \frac{5}{8}x$

Since the transverse axis endpoints have the same x-value, the hyperbola has a vertical transverse axis.

The endpoints of the transverse axis are also called the vertices of the hyperbola. The center of the hyperbola is at (h, k), which is the midpoint of the vertices, so:

- $(h, k) = \left(0, \frac{-10+10}{2}\right) = (0, 0)$
- $h = 0,\ k = 0$

The vertices of a hyperbola with a vertical transverse axis are $(h, k \pm a)$. So:

- $a = 10$

The asymptotes of a hyperbola with a vertical transverse axis are $y - k = \pm\frac{a}{b}(x - h)$. So:

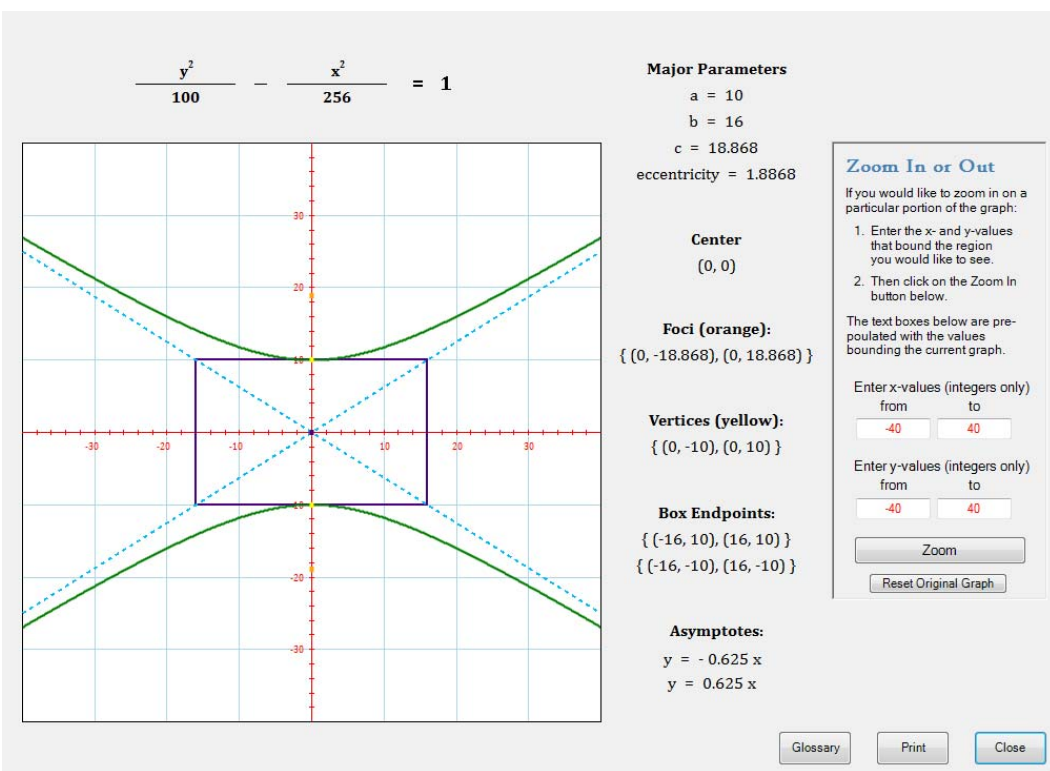
- $\frac{a}{b} = \frac{5}{8}$ and $a = 10$ so, $b = 16$

Standard form for a hyperbola with a vertical transverse axis is: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

So the standard form for the hyperbola defined above is:

$$\frac{y^2}{100} - \frac{x^2}{256} = 1$$

We are not required to graph the hyperbola, but here's what it looks like:



Convert the equation to the standard form for a parabola by completing the square on x or y as appropriate.

11) $y^2 - 4y + 8x + 44 = 0$

We will convert this equation to standard form as defined by Blitzer, which is

$$(y-k)^2 = 4p(x-h)$$

Original Equation:	$y^2 - 4y + 8x + 44 = 0$	
Subtract $(8x+44)$:	$y^2 - 4y$	$= -8x - 44$
Add $\left(\frac{4}{2}\right)^2 = 4$:	$y^2 - 4y + 4$	$= -8x - 40$
Simplify both sides:	$\boldsymbol{(y-2)^2 = -8(x+5)}$	

Find the standard form of the equation of the parabola using the information given.

12) Focus: (0, −2); Directrix: x = 6

The parabola described above has a vertical Directrix, so it opens left or right.

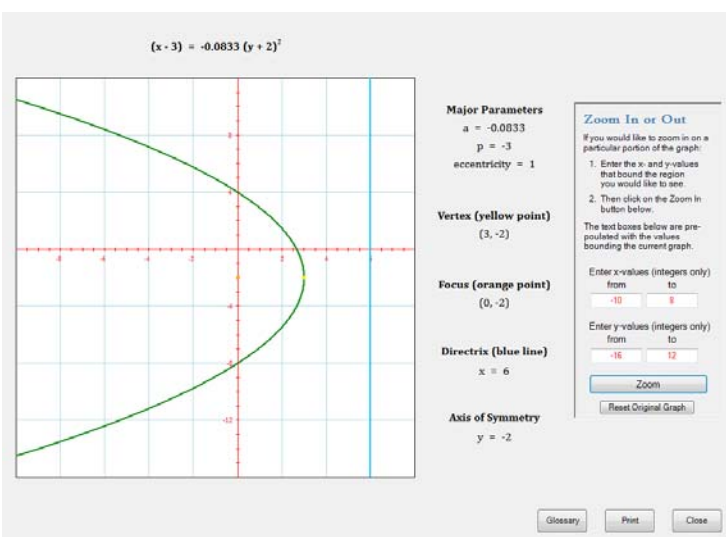
The focus is to the left of the Directrix, so the parabola opens to the left.

For a parabola with a vertical Directrix:

- The vertex is halfway between the focus and the Directrix, so the vertex is:
  $$(h,k) = \left(\frac{0+6}{2}, -2\right) = (3,-2) \quad \Rightarrow \quad h = 3;\ k = -2$$
- The focus is $(h+p, k) = (0,-2)$, so $h + p = 3 + p = 0$ and so, $p = -3$

We can now write the equation in what Blitzer defines as standard form: $(y-k)^2 = 4p(x-h)$

$$\boldsymbol{(y+2)^2 = -12(x-3)}$$



Eliminate the parameter t. Find a rectangular equation for the plane curve defined by the parametric equations.

13) $x = 9 \sin t$, $y = 9 \cos t$; $0 \le t \le 2\pi$

Since our equations exist in terms of the sine and cosine functions, let's take advantage of the trigonometric identity: $\sin^2 t + \cos^2 t = 1$.

Let's start with the equations given and modify them so we can use the identity.

$x = 9 \sin t$ $\qquad$ $y = 9 \cos t$

Square both equations and add them:

$x^2 + y^2 = 81 \sin^2 t + 81 \cos^2 t$

$x^2 + y^2 = 81(\sin^2 t + \cos^2 t)$

$\boldsymbol{x^2 + y^2 = 81}$ $\qquad$ This is a circle of radius 9.

Note that the values of x and y are both restricted; $|x| \le 9$, $|y| \le 9$.

A table of generic parametric equations from the Algebra (Main) App on www.mathguy.us is provided below (recall that for a parabola, $4ap = 1$):

Sample Parametric Equations for Conic Curves

Parabola	Circle
Cartesian Equation $(y-k) = a(x-h)^2$	**Cartesian Equation** $(x-h)^2 + (y-k)^2 = r^2$
Parametric Equations $x = h + t$, $y = k + at^2$	**Parametric Equations** $x = h + r\cos t$, $y = k + r\sin t$
Hyperbola	**Ellipse**
Cartesian Equation $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	**Cartesian Equation** $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
Parametric Equations $x = h + a\sec t$, $y = k + b\tan t$	**Parametric Equations** $x = h + a\cos t$, $y = k + b\sin t$

Graph the polar equation.

14) $r = \dfrac{8}{1 + 4\cos\theta}$ Identify the directrix and vertices.

The general polar equations of ellipses, parabolas and hyperbolas are all the same:

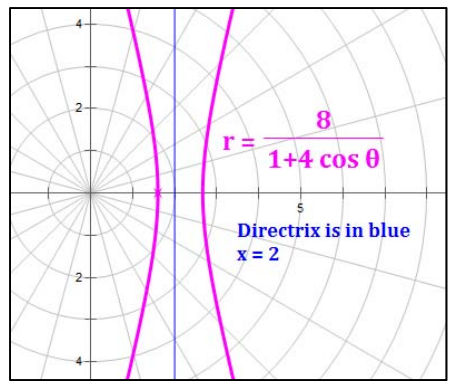
$$r = \frac{ep}{1 \pm e \cdot \cos\theta} \quad \text{or} \quad r = \frac{ep}{1 \pm e \cdot \sin\theta}$$

What varies among the curves is the value of the eccentricity, e, as follows:

- For an ellipse, $e = \frac{c}{a}$ and $e < 1$
- For a parabola, $e = 1$
- For a hyperbola, $e = \frac{c}{a}$ and $e > 1$

Now, let's look at the equation above:

- $r = \frac{8}{1+4\cdot\cos\theta}$
- e is the coefficient of the cosine function in the denominator, so $e = 4$. So, our curve is a hyperbola.
- The use of the cosine function implies the hyperbola has a horizontal transverse axis and vertical Directrixes.
- The numerator is $ep = 8$, so $4p = 8$, and so $p = 2$.



Then, **the Directrix requested by this problem is located $p = 2$ units to the right of the pole, (i.e., at $x = 2$).** Note: the Directrix is to the right of the pole because of the plus sign in the denominator.

Note: This curve has two foci and two Directrixes, but we were only asked to find one Directrix. The easiest one to find is the one based on the value of p.

In Polar form, the vertices occur at $\theta = 0$ and $\theta = \pi$.

- $f(0) = \dfrac{8}{1+4\cos(0)} = \dfrac{8}{5}$. So **one vertex is at $(r, \theta) = \left(\frac{8}{5}, 0\right)$**
- $f(\pi) = \dfrac{8}{1+4\cos(\pi)} = -\dfrac{8}{3}$. So **one vertex is at $(r, \theta) = \left(-\frac{8}{3}, \pi\right)$**

Parametric equations and a value for the parameter t are given. Find the coordinates of the point on the plane curve described by the parametric equations corresponding to the given value of t.

15) $x = t^3 + 1,\ y = 9 - t^4;\ t = 2$

Easy peezy. Just substitute the value of t into the equations for x and y.

When $t = 2$,

- $x = 2^3 + 1 = 9$
- $y = 9 - 2^4 = -7$
- The point in Cartesian form is $(\boldsymbol{x}, \boldsymbol{y}) = (\mathbf{9}, \mathbf{-7})$

16) $x = (50 \cos 30°)t,\ y = 9 + (50 \sin 30°)t - 18t^2;\ t = 6$

Easy peezy again. Just substitute the value of t into the equations for x and y.

When $t = 6$,

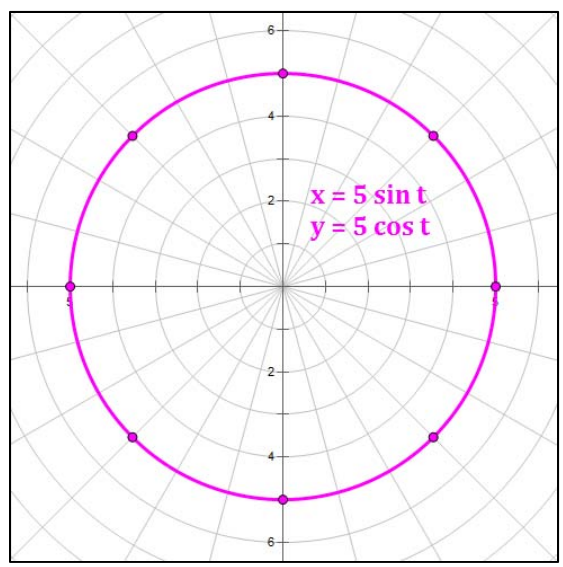
- $x = (50 \cos 30°) \cdot 6 = 50 \cdot \frac{\sqrt{3}}{2} \cdot 6 = 150\sqrt{3}$
- $y = 9 + (50 \sin 30°) \cdot 6 - 18(6)^2 = 9 + \left(50 \cdot \frac{1}{2} \cdot 6\right) - 648 = -489$
- The point in Cartesian form is $(\boldsymbol{x}, \boldsymbol{y}) = (\mathbf{150\sqrt{3}}, \mathbf{-489})$

Use point plotting to graph the plane curve described by the given parametric equations.

17) $x = 5 \sin t,\ y = 5 \cos t;\ 0 \le t \le 2\pi$

We may immediately recognize this as a circle of radius $r = 5$. However, let's plot some points:

t	x	y	Point
0	0	5	$(0, 5)$
$\pi/4$	$\frac{5\sqrt{2}}{2}$	$\frac{5\sqrt{2}}{2}$	$\left(\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right)$
$\pi/2$	5	0	$(5, 0)$
$3\pi/4$	$\frac{5\sqrt{2}}{2}$	$-\frac{5\sqrt{2}}{2}$	$\left(\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)$
π	0	-5	$(0, -5)$
$5\pi/4$	$-\frac{5\sqrt{2}}{2}$	$-\frac{5\sqrt{2}}{2}$	$\left(-\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)$
$3\pi/2$	-5	0	$(-5, 0)$
$7\pi/4$	$-\frac{5\sqrt{2}}{2}$	$\frac{5\sqrt{2}}{2}$	$\left(-\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right)$

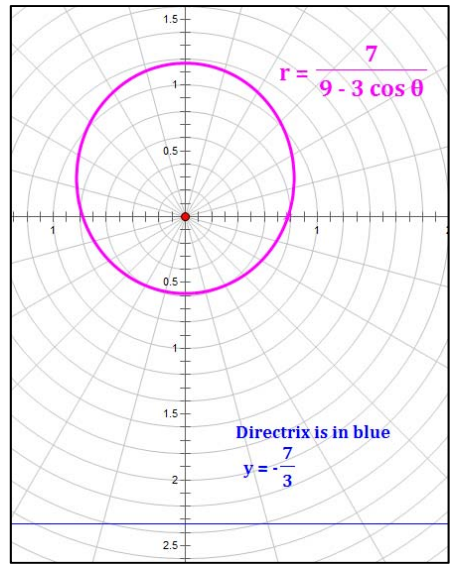


Identify the conic section that the polar equation represents. Describe the location of a directrix from the focus located at the pole.

18) $r = \dfrac{7}{9 - 3\sin\theta}$

Divide the numerator and denominator by 9 to obtain a lead value of 1 in the denominator:

$$r = \frac{ep}{1 - e\cdot\sin\theta} = \frac{(7/9)}{1 - (1/3)\cdot\sin\theta}$$



Then, $e = \frac{1}{3}$, so **the curve is an ellipse** because $e < 1$.

Now, let's look closer at the equation:

- The use of the sine function implies the ellipse has a vertical major axis and horizontal Directrixes.
- The numerator is $ep = \frac{7}{9}$, so $\frac{1}{3}p = \frac{7}{9}$, and so $p = \frac{7}{3}$.

Then, **the Directrix requested by this problem is located $p = \frac{7}{3}$ units below the pole (i.e., at $y = -\frac{7}{3}$).** Note: the Directrix is below the pole because of the minus sign in the denominator.

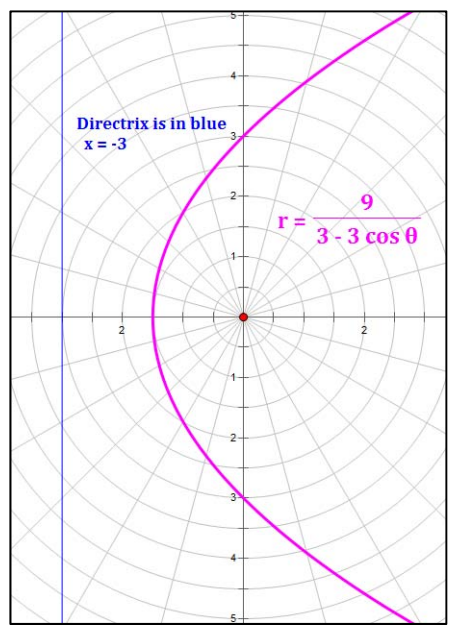
Note: This curve has two foci and two Directrixes, but we were only asked to find the Directrix associated with the focus at the pole.

Graph the polar equation.

19) $r = \dfrac{9}{3 - 3\cos\theta}$ Using the directrix and vertex.

Divide the numerator and denominator by 3 to obtain a lead value of 1 in the denominator:

$$r = \frac{ep}{1 - e\cdot\cos\theta} = \frac{3}{1 - 1\cdot\cos\theta}$$



Then, $e = 1$, so the curve is an parabola.

Now, let's look closer at the equation:

- The use of the cosine function implies the ellipse has a vertical Directrix, so it opens left or right.
- The minus sign in the denominator tells us that the parabola opens right.
- The numerator is $ep = 3$, so $1p = 3$, and so $p = 3$.

The Directrix, then, is at $x = -p \Rightarrow x = -3$.

The vertex is halfway between the pole and the Directrix, i.e., at $(x, y) = \left(-\frac{3}{2}, 0\right)$ in Cartesian coordinates, or at $(r, \theta) = \left(\frac{3}{2}, \pi\right)$ in Polar coordinates.

Find the standard form of the equation of the ellipse satisfying the given conditions.

20) Endpoints of major axis: (−2, 1) and (−2, 7); endpoints of minor axis: (−4, 4) and (0, 4)

Since the major axis endpoints have the same x-value, the ellipse has a vertical major axis.

An ellipse with a vertical major axis has the following major and minor axis vertices.

- The center is at (h, k), which is the midpoint of the major axis vertices, so:

$$(h,k) = \left(-2, \frac{1+7}{2}\right) = (-2, 4) \quad \Rightarrow \quad h = -2;\ \ k = 4$$

- Major axis vertices exist at $(h,\ k \pm a) = (-2,\ 4 \pm 3)$
- Minor axis vertices exist at $(h \pm b,\ k) = (-2 \pm 2,\ 4)$

From this we can determine that:

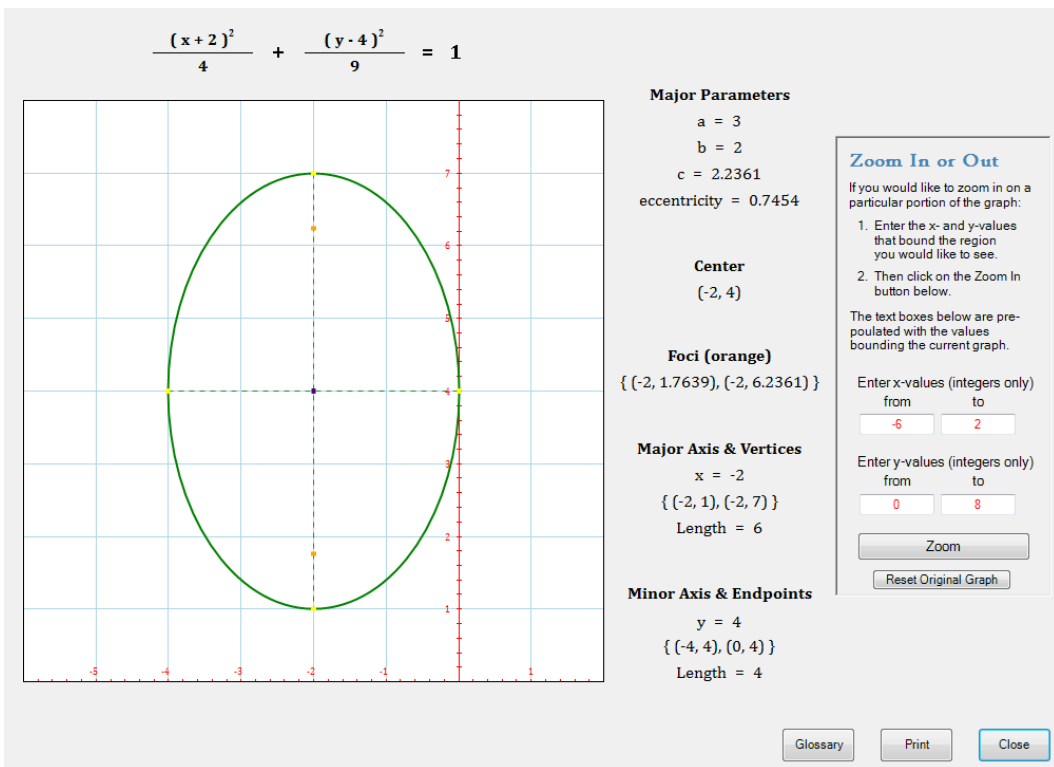
- $a = 3$
- $b = 2$

Standard form for an ellipse with a vertical major axis is: $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

So the standard form for the ellipse defined above is:

$$\frac{(x+2)^2}{4} + \frac{(y-4)^2}{9} = 1$$

We are not required to graph the ellipse, but here's what it looks like:



Find the vertex, focus, and directrix of the parabola with the given equation.

21) $(x + 1)^2 = -20(y - 3)$

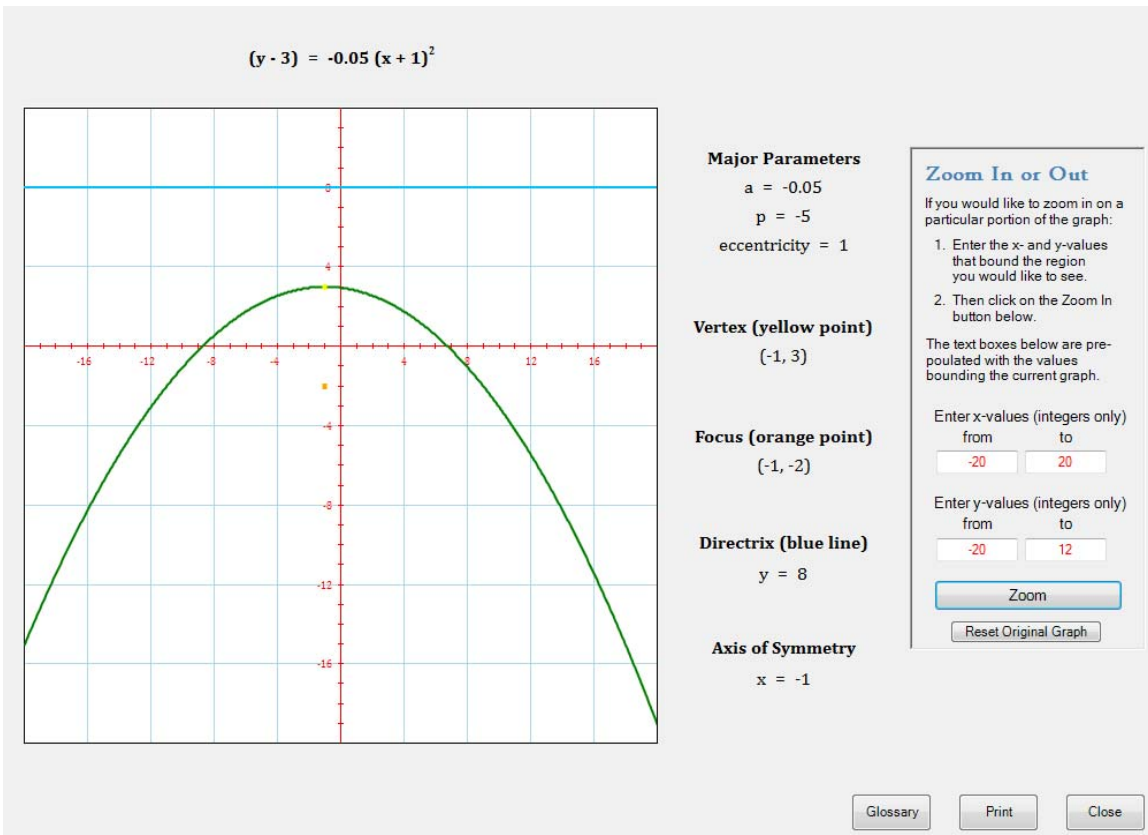
Consider the form of the above equation:

$$(x - h)^2 = 4p(y - k)$$

From this equation, we can determine the following:

- **The vertex of the parabola is $(h, k) = (-1, 3)$.**
- Since the x-term is squared, the parabola has a horizontal Directrix (i.e., it opens up or down).
- Since $4p = -20$, we conclude that $p = -5$, so the parabola opens down.
- **The focus of the parabola is at $(h, k + p) = (-1, 3 - 5) = (-1, -2)$.**
- **The Directrix of the parabola is at $y = k - p = 3 - (-5) = 8 \quad \Rightarrow \quad y = 8$.**

Okay. Although it is not required, let's see what this son of a gun looks like.



Solve the problem.

22) A baseball player hit a baseball with an initial velocity of 170 feet per second at an angle of 40° to the horizontal. The ball was hit at a height of 5 feet off the ground. Find parametric equations that describe the motion of the ball as a function of time. How long is the ball in the air? When is the ball at its maximum height? What is the horizontal distance the ball traveled?

Since we are dealing with gravity, the curve must be a parabola.

a) The parametric equations are as follows:

- Assuming no air resistance, the ball would travel in the x-direction at a constant velocity of $170\cos 40°$ feet per second, which produces the equation: $x = (170\cos 40°)t$
- In the y-direction, the ball begins 5 feet off the ground, has an initial velocity of $170\sin 40°$ feet per second and also has gravity acting upon it in the form of $-16t^2$. Together, these produce the equation: $y = 5 + (170\sin 40°)t - 16t^2$

The parametric equations, then, are:

$\boldsymbol{x = (170\cos 40°)t}$ or, roughly $\boldsymbol{x = 130.228\,t}$

$\boldsymbol{y = 5 + (170\sin 40°)t - 16t^2}$ or, roughly $\boldsymbol{y = -16t^2 + 109.274\,t + 5}$

b) The ball hits the ground when $y = 0$. So we can solve for t when $y = 0$.

$0 = -16t^2 + (170\sin 40°)\,t + 5$ Use the quadratic formula to solve for t.

$$t = \frac{-(170\sin 40°) \pm \sqrt{(170\sin 40°)^2 - 4(-16)(5)}}{2(-16)} = \{-0.04545407,\ 6.87507242\}$$

Using only the positive answer, we see that the ball was in the air: $\boldsymbol{t = 6.875}$ **seconds.**

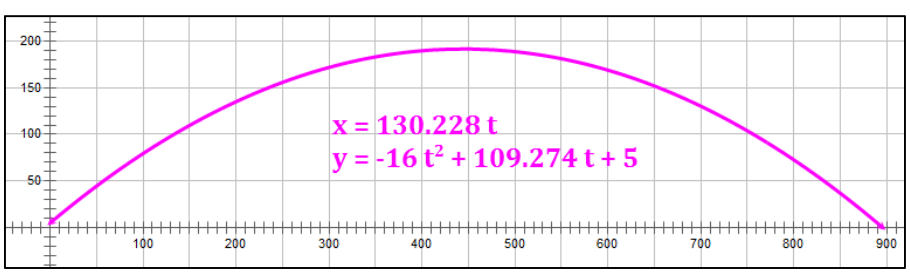
c) A parabola's maximum height occurs at the vertex, which occurs at $t = -\frac{b}{2a}$

So, maximum height occurs at $\boldsymbol{t} = \frac{-170\sin 40°}{2(-16)} = 3.414809 \sim \boldsymbol{3.415}$ **seconds.**

d) The distance the ball travelled (in the air) is the value of x when the ball hits the ground, i.e., at $t = 6.87507242$ seconds.

So, use the equation $x = (170\cos 40°)t$ to determine that the distance is:

$\boldsymbol{x} = 170\cos 40° \cdot 6.87507242 = \boldsymbol{895.324}$ **feet.**



Identify the conic section that the polar equation represents. Describe the location of a directrix from the focus located at the pole.

23) $r = \dfrac{3}{1 - 3\sin\theta}$

Using the standard Polar form for conic section curves,

$$r = \frac{ep}{1 - e\cdot\sin\theta} = \frac{3}{1 - 3\sin\theta}$$

Then, $e = 3$, so **the curve is a hyperbola** because $e > 1$.

Now, let's look closer at the equation:

- The use of the sine function implies the ellipse has a vertical transverse axis and horizontal Directrixes.
- The numerator is $ep = 3$, so $3p = 3$, and so $p = 1$.

Then, **the Directrix requested by this problem is located $p = 1$ unit below the pole (i.e., at $y = -1$).**
Note: the Directrix is below the pole because of the minus sign in the denominator.

Note: This curve has two foci and two Directrixes, but we were only asked to find the Directrix associated with the focus at the pole.