

Trigonometry

Function Relationships

$$\begin{aligned}\sin \theta &= \frac{1}{\csc \theta} & \csc \theta &= \frac{1}{\sin \theta} \\ \cos \theta &= \frac{1}{\sec \theta} & \sec \theta &= \frac{1}{\cos \theta} \\ \tan \theta &= \frac{1}{\cot \theta} & \cot \theta &= \frac{1}{\tan \theta} \\ \tan \theta &= \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{\cos \theta}{\sin \theta}\end{aligned}$$

Opposite Angle Formulas

$$\begin{aligned}\sin(-\theta) &= -\sin(\theta) \\ \cos(-\theta) &= \cos(\theta) \\ \tan(-\theta) &= -\tan(\theta) \\ \cot(-\theta) &= -\cot(\theta) \\ \sec(-\theta) &= \sec(\theta) \\ \csc(-\theta) &= -\csc(\theta)\end{aligned}$$

Cofunction Formulas (in Quadrant I)

$$\begin{aligned}\sin \theta &= \cos\left(\frac{\pi}{2} - \theta\right) & \cos \theta &= \sin\left(\frac{\pi}{2} - \theta\right) \\ \tan \theta &= \cot\left(\frac{\pi}{2} - \theta\right) & \cot \theta &= \tan\left(\frac{\pi}{2} - \theta\right) \\ \sec \theta &= \csc\left(\frac{\pi}{2} - \theta\right) & \csc \theta &= \sec\left(\frac{\pi}{2} - \theta\right)\end{aligned}$$

Pythagorean Identities

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ \tan^2 \theta + 1 &= \sec^2 \theta \\ \cot^2 \theta + 1 &= \csc^2 \theta\end{aligned}$$

Half Angle Formulas

$$\begin{aligned}\sin \frac{\theta}{2} &= \pm \sqrt{\frac{1 - \cos \theta}{2}} \\ \cos \frac{\theta}{2} &= \pm \sqrt{\frac{1 + \cos \theta}{2}} \\ \tan \frac{\theta}{2} &= \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \\ &= \frac{1 - \cos \theta}{\sin \theta} \\ &= \frac{\sin \theta}{1 + \cos \theta}\end{aligned}$$

Angle Addition Formulas

$$\begin{aligned}\sin(A + B) &= \sin A \cos B + \cos A \sin B \\ \sin(A - B) &= \sin A \cos B - \cos A \sin B \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \\ \tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ \tan(A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B}\end{aligned}$$

Double Angle Formulas

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= 1 - 2 \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \\ \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta}\end{aligned}$$

Product-to-Sum Formulas

$$\begin{aligned}\sin A \cdot \sin B &= \frac{1}{2} [\cos(A - B) - \cos(A + B)] \\ \cos A \cdot \cos B &= \frac{1}{2} [\cos(A - B) + \cos(A + B)] \\ \sin A \cdot \cos B &= \frac{1}{2} [\sin(A + B) + \sin(A - B)] \\ \cos A \cdot \sin B &= \frac{1}{2} [\sin(A + B) - \sin(A - B)]\end{aligned}$$

Triple Angle Formulas

$$\begin{aligned}\sin 3\theta &= 3 \sin \theta - 4 \sin^3 \theta \\ \cos 3\theta &= 4 \cos^3 \theta - 3 \cos \theta \\ \tan 3\theta &= \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}\end{aligned}$$

Power Reducing Formulas

$$\begin{aligned}\sin^2 \theta &= \frac{1 - \cos 2\theta}{2} \\ \cos^2 \theta &= \frac{1 + \cos 2\theta}{2} \\ \tan^2 \theta &= \frac{1 - \cos 2\theta}{1 + \cos 2\theta}\end{aligned}$$

Arc Length

$$S = r\theta$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos A \\ b^2 &= a^2 + c^2 - 2ac \cos B \\ c^2 &= a^2 + b^2 - 2ab \cos C\end{aligned}$$

Law of Tangents

$$\frac{a - b}{a + b} = \frac{\tan\left[\frac{1}{2}(A - B)\right]}{\tan\left[\frac{1}{2}(A + B)\right]}$$

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Euler's Formula

$$e^{i\theta} = \cos \theta + i \sin \theta = \operatorname{cis} \theta$$

DeMoivre's Formula

$$(r \operatorname{cis} \theta)^n = r^n \operatorname{cis} (n\theta)$$

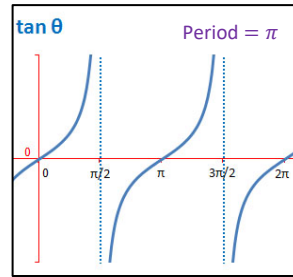
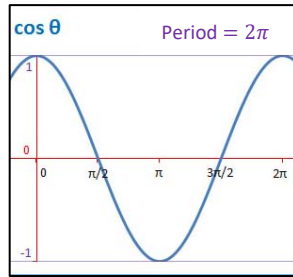
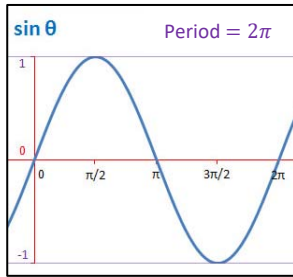
Polar Multiplication and Division

$$\begin{aligned}\text{Let: } a &= r_1 \operatorname{cis} \theta & b &= r_2 \operatorname{cis} \varphi \\ a \cdot b &= r_1 r_2 \operatorname{cis} (\theta + \varphi) & \frac{a}{b} &= \frac{r_1}{r_2} \operatorname{cis} (\theta - \varphi)\end{aligned}$$

Mollweide's Formulas

$$\begin{aligned}\frac{a + b}{c} &= \frac{\cos\left[\frac{1}{2}(A - B)\right]}{\sin\left(\frac{1}{2}C\right)} \\ \frac{a - b}{c} &= \frac{\sin\left[\frac{1}{2}(A - B)\right]}{\cos\left(\frac{1}{2}C\right)}\end{aligned}$$

Trigonometry



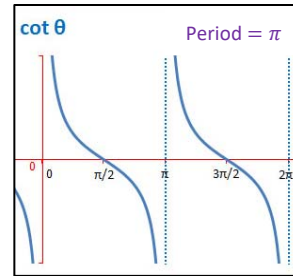
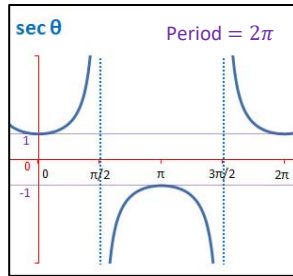
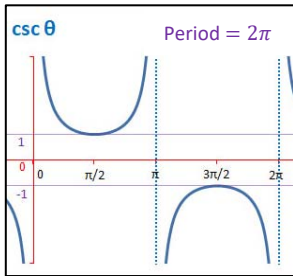
$y = A \cdot f(Bx - C) + D$

Amplitude: $|A|$

Period: $\frac{\text{parent "f" period}}{B}$

Phase Shift: $\frac{C}{B} \rightarrow$

Vertical Shift: D



Harmonic Motion

$d = a \cos \omega t$ or $d = a \sin \omega t$

$f = \frac{1}{\text{period}} = \frac{\omega}{2\pi}$

$\omega = 2\pi f, \omega > 0$

Trig Functions of Special Angles (Unit Circle)

θ Rad	θ°	$\sin \theta$	$\cos \theta$	$\tan \theta$
0	0°	0	1	0
$\pi/6$	30°	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
$\pi/4$	45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	90°	1	0	undefined

Signs of Trig Functions by Quadrant

$\sin \theta +$	$\sin \theta +$
$\cos \theta -$	$\cos \theta +$
$\tan \theta -$	$\tan \theta +$
$\sin \theta -$	$\sin \theta -$
$\cos \theta -$	$\cos \theta +$
$\tan \theta +$	$\tan \theta -$

Locations of Principal Values of Inverse Trig Functions

$\cos^{-1} \theta -$	$\sin^{-1} \theta +$
	$\cos^{-1} \theta +$
	$\tan^{-1} \theta +$
	$\sin^{-1} \theta -$
	$\tan^{-1} \theta -$

Rectangular/Polar Conversion

Rectangular	Polar
(x, y)	(r, θ)
$x = r \cos \theta$ $y = r \sin \theta$	$r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1} \left(\frac{y}{x} \right)$
$a + bi$	$r(\cos \theta + i \sin \theta)$ or $r \text{ cis } \theta$
$a = r \cos \theta$ $b = r \sin \theta$	$r = \sqrt{a^2 + b^2}$ $\theta = \tan^{-1} \left(\frac{b}{a} \right)$
$a\mathbf{i} + b\mathbf{j}$	$\ \mathbf{v}\ \angle \theta$
$a = \ \mathbf{v}\ \cos \theta$ $b = \ \mathbf{v}\ \sin \theta$	$\ \mathbf{v}\ = \sqrt{a^2 + b^2}$ $\theta = \tan^{-1} \left(\frac{b}{a} \right)$

Triangle Area

$A = \frac{1}{2}bh$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{1}{2}P = \frac{1}{2}(a+b+c)$

$A = \frac{1}{2} \left(\frac{a^2 \sin B \sin C}{\sin A} \right)$

$A = \frac{1}{2} ab \sin C$

$A = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$A = \frac{1}{2} \|\mathbf{u}\| \|\mathbf{v}\| \sin \theta$

Vector Properties

$0 + \mathbf{u} = \mathbf{u} + 0 = \mathbf{u}$

$\mathbf{u} + (-\mathbf{u}) = (-\mathbf{u}) + \mathbf{u} = 0$

$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$

$m(n\mathbf{u}) = (mn)\mathbf{u}$

$m(\mathbf{u} + \mathbf{v}) = m\mathbf{u} + m\mathbf{v}$

$(m+n)\mathbf{u} = m\mathbf{u} + n\mathbf{u}$

$1(\mathbf{v}) = \mathbf{v}$

$\|m\mathbf{v}\| = |m| \|\mathbf{v}\|$

Unit Vector: $\frac{\mathbf{v}}{\|\mathbf{v}\|}$

Vector Dot Product

$\mathbf{u} \cdot \mathbf{v} = (u_1 \cdot v_1) + (u_2 \cdot v_2)$

$\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \cdot \mathbf{v}) + (\mathbf{u} \cdot \mathbf{w})$

Vector Cross Product

$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{u}_1 & \mathbf{u}_2 \\ \mathbf{v}_1 & \mathbf{v}_2 \end{vmatrix} = u_1v_2 - u_2v_1$

$\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$

Angle between Vectors

$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$ $\sin \theta = \frac{\|\mathbf{u} \times \mathbf{v}\|}{\|\mathbf{u}\| \|\mathbf{v}\|}$

\perp iff $\mathbf{u} \cdot \mathbf{v} = 0$ \parallel iff $\mathbf{u} \times \mathbf{v} = 0$