Math Handbook

of Formulas, Processes and Tricks

(www.mathguy.us)

Pre-Algebra

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Blue = Developed specifically for Pre-Algebra Handbook

Green = Also included in Algebra Handbook

Purple = Also Included in Geometry Handbook

Cover art by Rebecca Williams,
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Useful Websites

Wolfram Math World – Perhaps the premier site for mathematics on the Web. This site contains definitions, explanations and examples for elementary and advanced math topics.

http://mathworld.wolfram.com/

Purple Math – A great site for the Algebra student, it contains lessons, reviews and homework guidelines. The site also has an analysis of your study habits. Take the Math Study Skills Self-Evaluation to see where you need to improve.

http://www.purplemath.com/

Math.com – Has a lot of information about Algebra, including a good search function.

http://www.math.com/homeworkhelp/Algebra.html

Algebra.com – Has short descriptions and demonstrations for a wide variety of Algebra topics.

http://www.algebra.com/

Math League – Specializes in math contests, books, and computer software for students from the 4th grade through high school.

http://www.mathleague.com/help/geometry/geometry.htm

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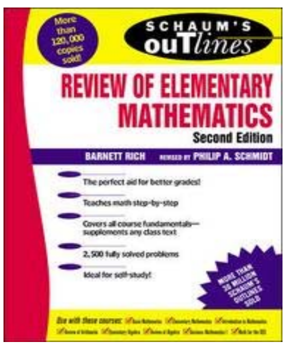
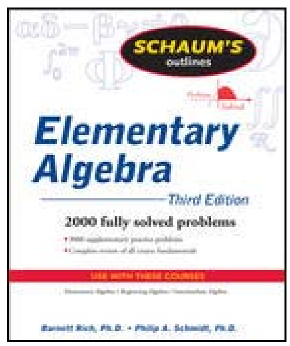
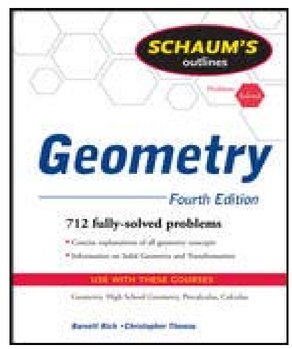
Schaum's Outlines

An important student resource for any high school math student is a Schaum's Outline. Each book in this series provides explanations of the various topics in the course and a substantial number of problems for the student to try. Many of the problems are worked out in the book, so the student can see examples of how they should be solved.

Schaum's Outlines are available at Amazon.com, Barnes & Noble, Borders and other booksellers.







Note: This study guide was prepared to be a companion to most books on the subject of High School Algebra. In particular, I used the following texts to determine which subjects to include in this guide.

- *Algebra 1* , by James Schultz, Paul Kennedy, Wade Ellis Jr, and Kathleen Hollowelly.
- *Algebra 2* , by James Schultz, Wade Ellis Jr, Kathleen Hollowelly, and Paul Kennedy.
- *Geometry* , by Ron Larson, Laurie Boswell, and Lee Stiff

Although a significant effort was made to make the material in this study guide original, some material from these texts was used in the preparation of the study guide.

Pre-Algebra
Divisibility Rules

The following rules can be used to determine whether a number is divisible by other numbers. This is particularly useful in reducing fractions to lowest terms because the rules can be used to test whether both the numerator and denominator are divisible by the same number.

n	**A number is divisible by "n" if and only if:**	**Examples**
2	It is even, i.e., if it ends in 0, 2, 4, 6 or 8.	16 (even because it end in a 6) 948 (even because it ends in an 8)
3	The sum of its digits is divisible by 3. You may apply this test multiple times if necessary.	42 (4+2=6) 948 (9+4+8=21, then 2+1=3)
4	The number formed by its last 2 digits is divisible by 4.	332 (32÷4=8) 1,908 (08÷4=2)
5	It ends in a 0 or 5.	905 (ends in a 5) 384,140 (ends in a 0)
6	It is divisible by both 2 and 3.	36 (it is even and 3+6=9) 948 (it is even and 9+4+8=21)
7	Double the last digit and subtract it from the rest of the number. If the result is divisible by 7, so is the original number. You may apply this test multiple times if necessary.	868 (86-[2·8]=70, and 70÷7=10) 2,345 (234-[2·5]=224, then apply again: 22-[2·4]=14, and 14÷7=2)
8	The number formed by its last 3 digits is divisible by 8.	92,104 (104÷8=13) 727,520 (520÷8=65)
9	The sum of its digits is divisible by 9. You may apply this test multiple times if necessary.	2,385 (2+3+8+5=18, then 1+8=9) 89,487 (8+9+4+8+7=36, then 3+6=9)
10	It ends in a 0.	370 (ends in a 0) 345,890 (ends in a 0)
11	The alternating sum and difference of its digits is divisible by 11.	374 (3-7+4=0) 9,482 (9-4+8-2=11)
12	It is divisible by both 3 and 4.	996 (9+9+6=24 and 96÷4=24) 1,344 (1+3+4+4=12 and 44÷4=11)

Note: $\mathbf{0}$ is divisible by every number except itself.

Pre-Algebra
Prime Numbers

Definitions

A prime number is a natural number (i.e., a positive integer) that has no factors other than 1 and itself. The prime numbers less than 50 are:

$$\mathbf{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47}$$

A composite number is a natural number that is not prime.

Prime Factorization

Every natural number has a unique prime factorization. This means that if you factor the number until all you have left are prime numbers, there is only one representation of the number in this form (ignoring the order of the factors). By mathematical convention, the prime factorization of a number is expressed as a product of its prime factors in numerical order, from low to high, with exponents on factors that are repeated.

Examples: $40 = 2^3 \cdot 5$ $\quad 330 = 2 \cdot 3 \cdot 5 \cdot 11$ $\quad 637 = 7^2 \cdot 13$

Deriving a Prime Factorization

Note: A number n can have at most one prime factor greater than $\sqrt{n}$.

To derive the unique prime factorization of a number n:

- Divide the number by 2 as many times as 2 will go into the number.
- Move up to the next prime number and repeat the process.
- Repeat the previous step until all of the factors are prime.

Examples: Find the prime factorizations of 336, 1000, and 2160.

$$\begin{aligned} 336 &= 2 \cdot 168 \\ &= 2 \cdot 2 \cdot 84 \\ &= 2 \cdot 2 \cdot 2 \cdot 42 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 21 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 7 \\ &= 2^4 \cdot 3 \cdot 7 \end{aligned}$$

$$\begin{aligned} 1000 &= 2 \cdot 500 \\ &= 2 \cdot 2 \cdot 250 \\ &= 2 \cdot 2 \cdot 2 \cdot 125 \\ &= 2 \cdot 2 \cdot 2 \cdot 5 \cdot 25 \\ &= 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 \\ &= 2^3 \cdot 5^3 \end{aligned}$$

$$\begin{aligned} 2160 &= 2 \cdot 1080 \\ &= 2 \cdot 2 \cdot 540 \\ &= 2 \cdot 2 \cdot 2 \cdot 270 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 135 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 45 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 15 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \\ &= 2^4 \cdot 3^3 \cdot 5 \end{aligned}$$

Pre-Algebra
Prime Factor Trees

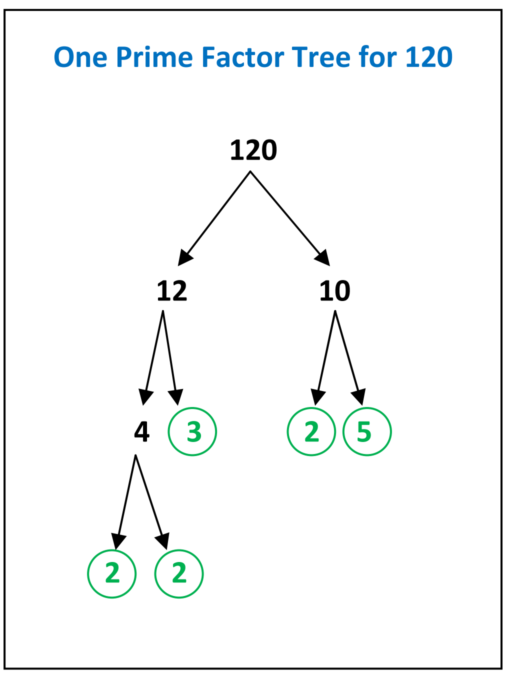
What is a Prime Factor Tree?

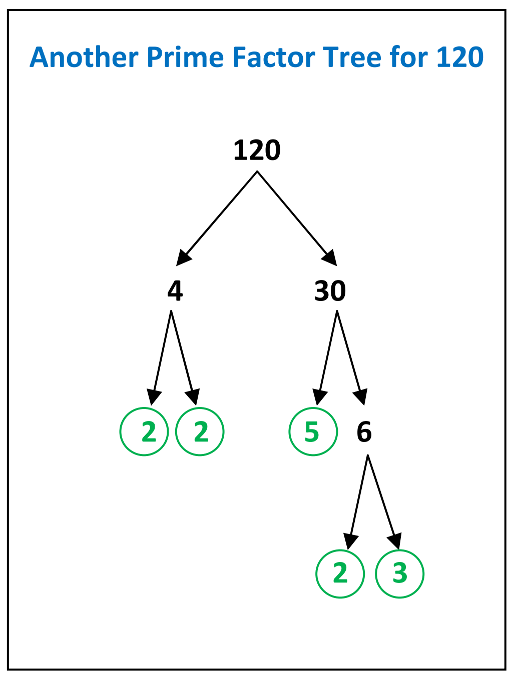
A **Prime Factor Tree** is a device that can be used to find the prime factors of a number. Even though each number has a unique set of prime factors, most numbers do not have a unique prime factor tree. The nice thing about a tree is that you can work with any factors of the number, and by the time you have finished, you have found its unique set of prime factors.

To develop a prime factor tree:

- Write the number to be factored at the top of the tree.
- Beneath the number, write a pair of factors that multiply to get the number.
- Repeat the above step until all of the factors are prime.
- It is useful to identify the prime factors you develop in some manner, like circling them.
- Collect all of the prime factors to obtain the prime factorization of the number.

Examples:





In both cases, the prime factorization of 120 is determined to be: $\mathbf{120 = 2^3 \cdot 3 \cdot 5}$

Notice that the two trees in the examples obtain the same result even though they take different paths to get that result. Other paths are possible as well. The important thing is the result, not the path.

Pre-Algebra
More about Prime Numbers

Interesting Facts about Primes

- 1 is defined to not be prime.
- 2 is the only even prime (2 divides every other even number).
- There are an infinite number of prime numbers.
- Large prime numbers form the basis of computer encryption routines used by banks and others on the internet.

Prime Testing

In order to determine if a number is prime, it is only necessary to try dividing it by primes less than its square root. That is because:

- If a composite number is a factor, then a smaller prime number is a factor.
- If a number larger than its square root is a factor, then a number less than its square root is a factor.

Example: Test whether 79 is prime.

Your first inclination might be to think you need to test every number less than 79 as a divisor of 79. However, from the above rule, it is only necessary to test 2, 3, 5, and 7; $\sqrt{79} < 9$, so only primes below 9 must be tested as factors. That's a lot less work and much more efficient use of your time. Since none of the factors divide 79, it is prime.

The Sieve of Eratosthenes

A method for finding prime numbers was invented by an ancient Greek mathematician from Cyrene named Eratosthenes. He created a table of numbers and began striking out multiples of prime numbers. Here is an example out to 25. If the table were larger, we would need to strike out multiples of more primes (all primes $\leq \sqrt{n}$, where n is the largest number in the table).

Strike out multiples of 2

	2,	3,	~~4~~,	5,
~~6~~,	7,	~~8~~,	9,	~~10~~,
11,	~~12~~,	13,	~~14~~,	15,
~~16~~,	17,	~~18~~,	19,	~~20~~,
21,	~~22~~,	23,	~~24~~,	25

Strike out multiples of 3

	2,	3,	~~4~~,	5,
~~6~~,	7,	~~8~~,	~~9~~,	~~10~~,
11,	~~12~~,	13,	~~14~~,	~~15~~,
~~16~~,	17,	~~18~~,	19,	~~20~~,
~~21~~,	~~22~~,	23,	~~24~~,	25

Strike out multiples of 5

	2,	3,	~~4~~,	5,
~~6~~,	7,	~~8~~,	~~9~~,	~~10~~,
11,	~~12~~,	13,	~~14~~,	~~15~~,
~~16~~,	17,	~~18~~,	19,	~~20~~,
~~21~~,	~~22~~,	23,	~~24~~,	~~25~~

The numbers remaining, after multiples of all primes are struck out, are prime.

Pre-Algebra
GCD and LCM

Simple methods for finding the Greatest Common Divisor (GCD) and the Least Common Multiple (LCM) are related, as shown below. Both involve developing a table of prime factors for the numbers in question. The methods are best illustrated by example.

Greatest Common Divisor (GCD)

Example A: Find the GCD of 180 and 105.

Step 1: Calculate the prime factors of each number and enter them into a small table:

180 =	2	x 2	x 3	x 3	x 5	
105 =			3		x 5	x 7
GCD =			3		x 5	

So, GCD = 3 x 5 = 15.

Step 2: Line up the prime factors so that those common to all of the numbers are in the same column.

Step 3: Bring any factors that show up for every number (i.e., that fill the column) below the line.

Step 4: Multiply all of the numbers below the line to obtain the GCD.

Example B: Find the GCD of 140, 210 and 462.

140 =	2	x 2		x 5	x 7	
210 =	2		x 3	x 5	x 7	
462 =	2		x 3		x 7	x 11
GCD =	2				x 7	

So, GCD = 2 x 7 = 14.

Example C: Find the GCD of 32 and 27.

32 =	2	x 2	x 2	x 2	x 2			
27 =						3	x 3	x 3

GCD = (there are no common factors)

So, GCD = 1.

Calculating a Number's Prime Factors

In order to calculate the prime factors of a number, simply begin dividing it by primes, starting with 2 and working higher until all factors are primes.

Examples: Find the prime factors of …

462	**180**
462 = 2 x 231	180 = 2 x 90
= 2 x 3 x 77	180 = 2 x 2 x 45
= 2 x 3 x 7 x 11	180 = 2 x 2 x 3 x 15
	180 = 2 x 2 x 3 x 3 x 5

If no common prime factors exist, GCD = 1 and the numbers are said to be ***relatively prime***.
Since 27 and 32 have no common prime factors, they are ***relatively prime***.

Pre-Algebra
GCD and LCM

Least Common Multiple (LCM)

Example A: Find the LCM of 12 and 18.

Step 1: Calculate the prime factors of each number and enter them into a small table:

12 =	2	x 2	x 3	
18 =	2		x 3	x 3
LCM =	2	x 2	x 3	x 3

So the LCM = 2 x 2 x 3 x 3 = 36

Step 2: Line up the prime factors so that those common to all of the numbers are in the same column.

Step 3: Bring one factor from every column below the line.

Step 4: Multiply all of the numbers below the line to obtain the LCM.

Example B: Find the LCM of 6, 8 and 18.

6 =	2			x 3	
8 =	2	x 2	x 2		
18 =	2			x 3	x 3
LCM =	2	x 2	x 2	x 3	x 3

So, the LCM = 2 x 2 x 2 x 3 x 3 = 72.

Lowest Common Denominator (LCD)

When fractions with different denominators are to be added or subtracted, it is necessary to find the Lowest Common Denominator. The LCD is essentially the Least Common Multiple of the denominators in question. Consider this problem:

Example: Calculate: $\frac{5}{12} - \frac{7}{18}$. In Example A, the LCM of 12 and 18 was calculated to be 36. To determine which fractional name for 1 must be multiplied by each fraction to obtain a common denominator, we look for the missing numbers in each row. From Example A above:

missing a "3"

12 =	2	x 2	x 3	□	
18 =	2	□	x 3	x 3	
LCM =	2	x 2	x 3	x 3	= 36

missing a "2"

The missing number in the row for 12 is 3. Therefore, we use $\frac{3}{3}$ as a multiplier for $\frac{5}{12}$. Similarly, we use $\frac{2}{2}$ as a multiplier for $\frac{7}{18}$.

Result: $\left(\frac{3}{3}\right)\left(\frac{5}{12}\right) - \left(\frac{7}{18}\right)\left(\frac{2}{2}\right) = \frac{15}{36} - \frac{14}{36} = \frac{1}{36}$

Pre-Algebra
Finding All Factors (Divisors)

Definition

A **factor** (also called a **divisor**) is a number that divides into another number, leaving no remainder.

Examples: 6 has the factors: $1, 2, 3, 6$.

30 has factors: $1, 2, 3, 5, 6, 10, 15, 30$.

Finding Factors

To find the factors of a number n

- Divide each number less than $\sqrt{n}$ into n to see if there is a remainder. Some can be tested quickly; for example, if the number is odd, no even numbers will divide it.
- Each successful division without remainder yields a pair of factors, one of which is less than $\sqrt{n}$ and one of which is greater than $\sqrt{n}$.
- Collect all of the factors, including both factors from each pair, into the solution set. Don't forget the "trivial factors" 1 and n.

Examples: Find all the factors of 28, 30, and 75.

$$\begin{aligned} 28 &= 1 \cdot 28 \\ &= 2 \cdot 14 \\ &= 4 \cdot 7 \end{aligned}$$

Factors $= \{1, 2, 4, 7, 14, 28\}$

$$\begin{aligned} 30 &= 1 \cdot 30 \\ &= 2 \cdot 15 \\ &= 3 \cdot 10 \\ &= 5 \cdot 6 \end{aligned}$$

Factors $= \{1, 2, 3, 5, 6, 10, 15, 30\}$

$$\begin{aligned} 75 &= 1 \cdot 75 \\ &= 3 \cdot 25 \\ &= 5 \cdot 15 \end{aligned}$$

Factors $= \{1, 3, 5, 15, 25, 75\}$

Perfect Numbers

A perfect number n is a number whose factors, including 1 but excluding n, add to n. For example, see the factors of 28 in the example above $(1 + 2 + 4 + 7 + 14 = 28)$. The first six perfect numbers are:

$$6 \quad 28 \quad 496 \quad 8{,}128 \quad 33{,}550{,}336 \quad 8{,}589{,}869{,}056$$

- All even perfect numbers except 6 have the remainder 1 when divided by 9.
- It is not known whether there are any odd perfect numbers.

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Pre-Algebra
Finding All Factors, a Second Approach

Another way to find all of the factors of a number is to work with its prime factorization. First recall that every number has a unique prime factorization. Next consider what the prime factorization of the number says about its factors.

- Each factor must be the product of a subset of the prime factors of the number.
- Each possible subset of prime factors will generate a unique factor of the number.

Finding Factors with Prime Factorization

Let $\{p_1, p_2, \ldots, p_k\}$ be the set of primes included in the prime factorization of a number N.

Let $\{a_1, a_2, \ldots, a_k\}$ be the exponents of those primes in the prime factorization of N.

Then, the prime factorization of N can be expressed as:

$$\boldsymbol{N = {p_1}^{a_1} \cdot {p_2}^{a_2} \cdot \ldots \cdot {p_k}^{a_k}}$$

Factors can be formed from the products of each possible combination of primes in this formula. Note that each prime $\boldsymbol{p_i}$ can occur as few as zero times or as many as $\boldsymbol{a_i}$ times, a total of $(\boldsymbol{a_i} + \mathbf{1})$ possibilities. The number of factors of N, then, is given by:

$$\boldsymbol{\tau(N) = (a_1 + 1) \cdot (a_2 + 1) \cdot \ldots \cdot (a_k + 1)}$$

Note: "$\boldsymbol{\tau}$" (pronounced "tau") is the Greek letter corresponding to the English "$\boldsymbol{t}$"; it is used here to represent the "$\boldsymbol{t}$"otal number of factors of N.

Example: *Find all factors of the number 60.*

First note that: $\mathbf{60 = 2^2 \cdot 3^1 \cdot 5^1}$

Then, $\boldsymbol{\tau}(\mathbf{60}) = [(\mathbf{2+1}) \cdot (\mathbf{1+1}) \cdot (\mathbf{1+1})] = \mathbf{12}$. Therefore, the number 60 has 12 factors.

What are the 12 factors?

$2^0 \cdot 3^0 \cdot 5^0 = 1$	$2^1 \cdot 3^0 \cdot 5^0 = 2$	$2^2 \cdot 3^0 \cdot 5^0 = 4$
$2^0 \cdot 3^1 \cdot 5^0 = 3$	$2^1 \cdot 3^1 \cdot 5^0 = 6$	$2^2 \cdot 3^1 \cdot 5^0 = 12$
$2^0 \cdot 3^0 \cdot 5^1 = 5$	$2^1 \cdot 3^0 \cdot 5^1 = 10$	$2^2 \cdot 3^0 \cdot 5^1 = 20$
$2^0 \cdot 3^1 \cdot 5^1 = 15$	$2^1 \cdot 3^1 \cdot 5^1 = 30$	$2^2 \cdot 3^1 \cdot 5^1 = 60$

In summary, the factors of 60 are: $\mathbf{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}$

Pre-Algebra
Roman Numerals

Roman Numerals are a system of numbers developed in Ancient Rome. Interestingly, there is no Roman Numeral for zero; they start at 1.

The symbols used and their corresponding values are:

Symbol	Value
I	1
V	5
X	10
C	100
D	500
M	1,000

Larger numbers are created by placing a bar over a numeral; this indicates it should be multiplied by 1,000.

Symbol	Value
$\overline{\text{V}}$	5,000
$\overline{\text{X}}$	10,000
$\overline{\text{C}}$	100,000
$\overline{\text{D}}$	500,000
$\overline{\text{M}}$	1,000,000

Numerals are expressed by combining symbols according to the following set of rules:

- They are written with the largest symbols to the left.
- Smaller symbols written to the right indicate addition (e.g., DC indicates 600).
- A smaller number written to the left indicates subtraction (e.g., IV indicates 4).
- No more than three of a symbol can be used in a numeral (e.g., 30 is XXX, 40 is XL).
- If a digit is formed in a particular way, it is also formed that way in larger numerals, e.g., 99 is XCIX which is 90 (XC) plus 9(IX), not IC (100-1).

Roman Numerals for Key Values

1	I	10	X	100	C
2	II	20	XX	200	CC
3	III	30	XXX	300	CCC
4	IV	40	XL	400	CD
5	V	50	L	500	D
6	VI	60	LX	600	DC
7	VII	70	LXX	700	DCC
8	VIII	80	LXXX	800	DCCC
9	IX	90	XC	900	CM
1,000	M	2,000	MM	3,000	MMM

Combine the numerals at left to make any number from 1 to 3,999.

Examples:

32 = XXXII

476 = CDLXXVI

514 = DXIV

888 = DCCCLXXXVIII

999 = CMXCIX

2008 = MMVIII

Pre-Algebra
Metric Measures

King Henry

The following mnemonic device can be used to remember the order of metric measurements:

	King	Henry	Died	By	Drinking	Chocolate	Milk
Length	km	hm	dkm	meters	dm	cm	mm
Mass	kg	hg	dkg	grams	dg	cg	mg
Volume	kl	hl	dkl	liters	dl	cl	ml

$\longleftarrow$

In moving from right to left:

For each position traversed,

- Divide by 10, or
- Move the decimal one place to the left.
- Add zeroes if needed.

$\longrightarrow$

In moving from left to right:

For each position traversed,

- Multiply by 10, or
- Move the decimal one place to the right.
- Add zeroes if needed.

In the mnemonic, the "b" in "by" stands for "base unit"; this is the unit that all others are based upon. The base units above are meters, grams, and liters.

Note: 1 ml = 1 cubic centimeter

The prefixes to the base unit, along with their meanings are:

$k\ =\ kilo$	$1{,}000$
$h = hecto$	100
$dk = deka$	10

$d = deci$	$\frac{1}{10}$
$c = centi$	$\frac{1}{100}$
$m = milli$	$\frac{1}{1{,}000}$

Examples:

$1\ km = 100{,}000\ cm$	Add 5 zeroes to the right (for 5 positions moved in the above chart).
$32\ mg = .000\ 032\ kilograms$	Move the decimal 6 places to the left (for the 6 positions moved to the left in the above chart).
$2.5\ liters = 2{,}500\ ml$	Move the decimal 3 places to the right (for the 3 positions moved to the right in the above chart).

Pre-Algebra
Measures and Weights – U.S. Conversions

Distance
1 foot = 12 inches
1 yard = 3 feet = 36 inches
1 fathom = 2 yards = 6 feet
1 rod = 5.5 yards = 16.5 feet
1 furlong = 40 rods = 220 yards
1 mile = 8 furlongs = 1,760 yards = 5,280 feet
1 league = 3 miles = 24 furlongs
1 acre = 43,560 square feet
1 square mile = 640 acres

Weight
1 pennyweight = 24 grains
1 dram = 27.344 grains
1 ounce = 16 drams = 437.5 grains
1 Troy ounce = 20 pennyweight = 480 grains
1 pound = 16 ounces = 7,000 grains
1 stone = 14 pounds
1 ton = 2,000 pounds
1 long ton = 2,240 pounds

Time
1 minute = 60 seconds
1 hour = 60 minutes = 3,600 seconds
1 day = 24 hours = 1,440 minutes
1 week = 7 days = 168 hours
1 fortnight = 2 weeks = 14 days
1 month = 4 1/3 weeks
1 year = 12 months = 52 weeks
1 year = 365 1/4 days

Capacity
1 fluid dram = 60 minims
1 fluid ounce = 8 fluid drams
1 gill = 4 fluid ounces
1 cup = 2 gills = 8 fluid ounces
1 pint = 2 cups = 16 fluid ounces
1 quart = 2 pints = 4 cups
1 gallon = 4 quarts = 16 cups
1 peck = 2 gallons
1 bushel = 4 pecks = 8 gallons

Pre-Algebra
Measures and Weights – U.S./Metric Conversions

U.S. to Metric

Distance
1 millimeter = .03937 inch
1 centimeter = .3937 inch
1 meter = 3.281 feet
1 kilometer = .6214 mile

Weight
1 carat = 3.086 grains
1 gram = .03527 ounces
1 kilogram = 2.2046 pounds
1 metric ton = 2,204.6 pounds

Capacity
1 milliliter = 16.23 minims
1 milliliter = .03381 fluid ounce
1 liter = 1.0567 quarts
1 liter = .2642 gallon

Metric to U.S.

Distance
1 inch = 2.54 centimeters
1 foot = 30.48 centimeters
1 yard = .9144 meter
1 mile = 1.6093 kilometers

Weight
1 ounce = 28.35 grams
1 pound = .4536 kilogram
1 ton = .9072 metric ton
1 long ton = 1.016 metric tons

Capacity
1 fluid ounce = 29.57 milliliters
1 pint = 473.2 milliliters
1 quart = .9464 liter
1 gallon = 3.785 liters

Other Metric Weights:

- 1 carat = 200 milligrams (5 carats = 1 gram)
- 1 metric ton = 1,000 kilograms

Pre-Algebra
Order of Operations

To the non-mathematician, there may appear to be multiple ways to evaluate an algebraic expression. For example, how would one evaluate the following?

$$3 \cdot 4 \cdot 7 + 6 \cdot 5^2$$

You could work from left to right, or you could work from right to left, or you could do any number of other things to evaluate this expression. As you might expect, mathematicians do not like this ambiguity, so they developed a set of rules to make sure that any two people evaluating an expression would get the same answer.

PEMDAS

In order to evaluate expressions like the one above, mathematicians have defined an order of operations that must be followed to get the correct value for the expression. The acronym that can be used to remember this order is **PEMDAS**. Alternatively, you could use the mnemonic phrase "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally" or make up your own way to memorize the order of operations. The components of **PEMDAS** are:

- **P** Anything in **P**arentheses is evaluated first.
- **E** Items with **E**xponents are evaluated next.
- **M** **M**ultiplication and …
- **D** **D**ivision are performed next.
- **A** **A**ddition and …
- **S** **S**ubtraction are performed last.

Usually when there are multiple operations in the same category, for example 3 multiplications, they can be performed in any order, but it is easiest to work from left to right.

Parenthetical Device. A useful device is to use apply parentheses to help you remember the order of operations when you evaluate an expression. Parentheses are placed around the items highest in the order of operations; then solving the problem becomes more natural. Using PEMDAS and this parenthetical device, we solve the expression above as follows:

Initial Expression:	$3 \cdot 4 \cdot 7 + 6 \cdot 5^2$
Add parentheses/brackets:	$= (3 \cdot 4 \cdot 7) + [6 \cdot (5^2)]$
Solve using PEMDAS:	$= (84) + (6 \cdot 25)$
	$= (84) + (150)$
Final Answer	$= 234$

Note: Any expression which is ambiguous, like the one above, is poorly written. Students should strive to ensure that any expressions they write are easily understood by others and by themselves. Use of parentheses and brackets is a good way to make your work more understandable.

Pre-Algebra
Properties of Algebra

Properties of Addition and Multiplication. For any real numbers **a**, **b**, and **c**:

Property	Definition for Addition	Definition for Multiplication
Closure Property	$a+b$ is a real number	$a \cdot b$ is a real number
Identity Property	$a+0=0+a=a$	$a \cdot 1 = 1 \cdot a = a$
Inverse Property	$a+(-a)=(-a)+a=0$	$For\ a \neq 0,\ \ a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$
Commutative Property	$a+b=b+a$	$a \cdot b = b \cdot a$
Associative Property	$(a+b)+c=a+(b+c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Distributive Property	$a \cdot (b+c) = (a \cdot b)+(a \cdot c)$	

Properties of Zero. For any real number **a**:

Multiplication by 0	$a \cdot 0 = 0 \cdot a = 0$
0 Divided by Something	$For\ a \neq 0,\ \frac{0}{a} = 0$
Division by 0	$\frac{a}{0}$ is undefined (even if $a = 0$)

Properties of Equality. For any real numbers **a**, **b**, and **c**:

Property	Definition
Addition Property	$If\ a=b, then\ a+c=b+c$
Subtraction Property	$If\ a=b, then\ a-c=b-c$
Multiplication Property	$If\ a=b, then\ a \cdot c = b \cdot c$
Division Property	$If\ a=b\ and\ c \neq 0, then\ a \div c = b \div c$

Pre-Algebra
Linear Patterns

Recognizing Linear Patterns

The first step to recognizing a pattern is to arrange a set of numbers in a table. The table can be either horizontal or vertical. Here, we consider the pattern in a horizontal format. More advanced analysis generally uses the vertical format.

Consider this pattern:

x-value	0	1	2	3	4	5
y-value	6	9	12	15	18	21

To analyze the pattern, we calculate differences of successive values in the table. These are called first differences. If the first differences are constant, we can proceed to converting the pattern into an equation. If not, we do not have a linear pattern. In this case, we may choose to continue by calculating differences of the first differences, which are called second differences, and so on until we get a pattern we can work with.

In the example above, we get a constant set of first differences, which tells us that the pattern is indeed linear.

x-value	0	1	2	3	4	5
y-value	6	9	12	15	18	21

First Differences	3	3	3	3	3

Converting a Linear Pattern to an Equation

Creating an equation from the pattern is easy if you have constant differences and a y-value for x = 0. In this case,

- The equation takes the form $\boldsymbol{y = mx + b}$, where
- ***"m"*** is the constant difference from the table, and
- ***"b"*** is the y-value when x = 0.

In the example above, this gives us the equation: $\boldsymbol{y = 3x + 6}$.

Note: If the table does not have a value for x=0, you can still obtain the value of **"b"**. Simply extend the table left or right until you have an x-value of 0; then use the first differences to calculate what the corresponding y-value would be. This becomes your value of **"b"**.

Finally, it is a good idea to test your equation. For example, if $x = 4$, the above equation gives $y = (3 \cdot 4) + 6 = 18$, which is the value in the table. So we can be pretty sure our equation is correct.

Pre-Algebra
Operating with Real Numbers

Absolute Value

The absolute value of something is the distance it is from zero. The easiest way to get the absolute value of a number is to eliminate its sign. Absolute values are always positive or 0.

$$|-5| = 5 \qquad |3| = 3 \qquad |0| = 0 \qquad \left|-\frac{3}{4}\right| = \frac{3}{4} \qquad |1.5| = 1.5$$

Adding and Subtracting Real Numbers

Adding Numbers with the Same Sign:

- Add the numbers without regard to sign.
- Give the answer the same sign as the original numbers.
- Examples:

$$(-6) + (-3) = -9$$
$$12 + 6 = 18$$

Adding Numbers with Different Signs:

- Ignore the signs and subtract the smaller number from the larger one.
- Give the answer the sign of the number with the greater absolute value.
- Examples:

$$(-6) + 3 = -3$$
$$(-7) + 11 = 4$$

Subtracting Numbers:

- Change the sign of the number or numbers being subtracted.
- Add the resulting numbers.
- Examples:

$$(-6) - (-3) = (-6) + 3 = -3$$
$$13 - 4 = 13 + (-4) = 9$$

Multiplying and Dividing Real Numbers

Numbers with the Same Sign:

- Multiply or divide the numbers without regard to sign.
- Give the answer a "+" sign.
- Examples:

$$(-6) \cdot (-3) = +18 = 18$$
$$12 \div 3 = +4 = 4$$

Numbers with Different Signs:

- Multiply or divide the numbers without regard to sign.
- Give the answer a "-" sign.
- Examples:

$$(-6) \cdot (3) = -18$$
$$12 \div (-3) = -4$$

Pre-Algebra
Adding and Subtracting Fractions

To add or subtract fractions:

- Rewrite the problem if necessary, in order to make it easier to work.
- Calculate a common denominator. *Note: another page shows how to do this.*
- Express each fraction in terms of the common denominator.
- Add or subtract the numerators of the fractions. Leave the denominator unchanged.
- Simplify if possible. Note: if you want to convert the solution to a mixed number, first simplify the fraction resulting from the addition or subtraction; then, calculate the mixed number.

Example 1: $\frac{3}{8}+\frac{11}{12}$

Create LCD: $=\frac{3}{3}\cdot\frac{3}{8}+\frac{11}{12}\cdot\frac{2}{2}$

Express with LCD: $=\frac{9}{24}+\frac{22}{24}$

Add numerators: $=\frac{31}{24}$

Simplify: $=1\,\frac{7}{24}$

Example 2: $\frac{1}{3}-\frac{2}{5}$

Create LCD: $=\frac{5}{5}\cdot\frac{1}{3}-\frac{2}{5}\cdot\frac{3}{3}$

Express with LCD: $=\frac{5}{15}-\frac{6}{15}$

Add numerators: $=\frac{-1}{15}$

Simplify: $=-\frac{1}{15}$

Example 3: $\left(-\frac{1}{9}\right)-\left(\frac{-5}{6}\right)$

Rewrite problem: $\left(-\frac{1}{9}\right)+\left(\frac{5}{6}\right)$

Create LCD: $=\frac{2}{2}\cdot\frac{(-1)}{9}+\frac{5}{6}\cdot\frac{3}{3}$

Express with LCD: $=\frac{-2}{18}+\frac{15}{18}$

Add numerators: $=\frac{13}{18}$

Example 4: $-\frac{4}{7}+\frac{9}{28}$

Create LCD: $=\frac{4}{4}\cdot\frac{(-4)}{7}+\frac{9}{28}$

Express with LCD: $=\frac{-16}{28}+\frac{9}{28}$

Add numerators: $=\frac{-7}{28}$

Simplify: $=-\frac{1}{4}$

Pre-Algebra
Multiplying and Dividing Fractions

To multiply or divide fractions:

- Rewrite the problem if necessary, in order to make it easier to work.
- If the problem is a division, invert the divisor and change the sign to multiplication ("flip that guy and multiply").
- Bring the numerators and denominators together in a single fraction.
- Simplify the multiplications you will need to do by reducing the factors in the numerator and denominator.
- Multiply both the numerators and denominators of the fractions.
- Simplify if possible. Note: if you want to convert the solution to a mixed number, first simplify the fraction resulting from the multiplication; then, calculate the mixed number.

Example 1: $\frac{2}{3} \cdot \frac{9}{14}$

Bring together: $= \frac{2 \cdot 9}{3 \cdot 14}$

Reduce by 2's: $= \frac{1 \cdot 9}{3 \cdot 7}$

Reduce by 3's: $= \frac{1 \cdot 3}{1 \cdot 7}$

Multiply: $= \frac{3}{7}$

Example2: $\frac{4}{7} \div \frac{19}{21}$

Flip divisor: $= \frac{4}{7} \cdot \frac{21}{19}$

Bring together: $= \frac{4 \cdot 21}{7 \cdot 19}$

Reduce by 7's: $= \frac{4 \cdot 3}{1 \cdot 19}$

Multiply: $= \frac{12}{19}$

Example 3: $\left(-\frac{1}{24}\right) \div \left(\frac{-5}{32}\right)$

Flip + Rewrite: $= \left(\frac{1}{24}\right) \cdot \left(\frac{32}{5}\right)$

Bring together: $= \frac{1 \cdot 32}{24 \cdot 5}$

Reduce by 8's: $= \frac{1 \cdot 4}{3 \cdot 5}$

Multiply: $= \frac{4}{15}$

Example 4: $\left(-\frac{6}{17}\right) \cdot \frac{5}{12}$

Rewrite: $= \frac{-6}{17} \cdot \frac{5}{12}$

Bring together: $= \frac{(-6) \cdot 5}{17 \cdot 12}$

Reduce by 6's: $= \frac{(-1) \cdot 5}{17 \cdot 2}$

Multiply: $= -\frac{5}{34}$

Pre-Algebra
Mixed Numbers and Improper Fractions

Definitions

A **proper fraction** is one in which (ignoring the sign) the numerator is less than the denominator. Proper fractions are between -1 and 1. It need not be in lowest terms.

An **improper fraction** is one in which (ignoring the sign) the numerator is greater than or equal to the denominator. Proper fractions are less than -1 or greater than 1. It need not be in lowest terms. Remember that being less than -1 means "more negative" than -1. So, for example, -2 and $-\frac{7}{5}$ are both less than -1.

A **mixed number** is a whole number followed by a proper fraction.

Examples:

Proper Fractions	$\frac{2}{3}, \ \frac{6}{7}, \ -\frac{1}{9}, \ \frac{375}{401}, \ -\frac{18}{20}$
Improper Fractions	$-\frac{4}{4}, \ \frac{17}{12}, \ -\frac{29}{2}, \ \frac{97}{96}, \ \frac{375}{25}$
Mixed Numbers	$2\frac{3}{4}, \ -6\frac{1}{2}, \ 13\frac{12}{21}, \ -1\frac{4}{7}, \ 365\frac{1}{4}$

Lowest Terms

A fraction is in **lowest terms** when it does not share any factors in the numerator and denominator. To put a fraction in lowest terms divide any common divisors out of both the numerator and denominator. **Example:** $\frac{14}{22} = \frac{7}{11}$ when we divide the common factor 2 out of both the numerator and denominator.

Conversion between Mixed Numbers and Improper Fractions

Mixed Number to Improper Fraction:

Multiply the whole number by the denominator of the fraction and add it to the numerator of the fraction to get the numerator of the improper fraction:

$$2\frac{3}{4} = \frac{(2 \cdot 4) + 3}{4} = \frac{11}{4}$$

Improper Fraction to Mixed Number:

Divide the numerator by the denominator. Show the result as a whole number followed by the remainder expressed as a fraction:

$$\frac{11}{4} = 11 \div 4 = 2\ r\ 3 = 2\frac{3}{4}$$

Pre-Algebra
Adding and Subtracting Mixed Numbers

To add or subtract mixed numbers,

- Rearrange the terms so the whole numbers are together and the fractions are together.
- Add or subtract the whole numbers.
- Add or subtract the fractional parts.
- Add the results of the first two steps. If subtracting, you may need to borrow 1 from the whole numbers in order to perform the subtraction.
- Simplify, if possible.

Example 1: $2\frac{6}{11} + 4\frac{4}{33}$

Rearrange terms: $= (2+4) + \left(\frac{6}{11} + \frac{4}{33}\right)$

Add whole numbers: $= 6 + \left(\frac{3}{3}\cdot\frac{6}{11} + \frac{4}{33}\right)$

Add fractions: $= 6 + \left(\frac{18+4}{33}\right)$

Recombine, Simplify: $= 6\frac{22}{33} = \mathbf{6\frac{2}{3}}$

Example 2: $8\frac{3}{14} - 2\frac{8}{21}$

Rearrange terms: $= (8-2) + \left(\frac{3}{14} - \frac{8}{21}\right)$

Subtract whole numbers: $= 6 + \left(\frac{3}{3}\cdot\frac{3}{14} - \frac{8}{21}\cdot\frac{2}{2}\right)$

Subtract fractions: $= 6 + \left(\frac{9-16}{42}\right) = 6 + \frac{-7}{42}$

Borrow 1 for subtraction: $= 5 + \left(1 - \frac{7}{42}\right) = 5 + \left(\frac{42-7}{42}\right)$

Simplify: $= 5\frac{35}{42} = \mathbf{5\frac{5}{6}}$

Pre-Algebra
Multiplying Mixed Numbers

The two methods shown below are equivalent. Use whichever one you like best.

Box Method

In the box method,

- Create a 2x2 array of multiplications from the parts of the fractions.
- Perform the 4 multiplications.
- Add the results.

Example: Multiply $\left(2\frac{3}{7}\right) \cdot \left(4\frac{2}{5}\right)$

Multiply	**2**	$\mathbf{\frac{3}{7}}$
4	8	$\frac{12}{7}$
$\mathbf{\frac{2}{5}}$	$\frac{4}{5}$	$\frac{6}{35}$

The result is obtained by adding the results of the 4 separate multiplications.

$$\left(2\frac{3}{7}\right) \cdot \left(4\frac{2}{5}\right) = 8 + \frac{12}{7} + \frac{4}{5} + \frac{6}{35}$$

$$= 8 + \frac{60 + 28 + 6}{35}$$

$$= 8 + \frac{94}{35}$$

$$= 8 + 2\frac{24}{35}$$

$$= \mathbf{10\frac{24}{35}}$$

Improper Fraction Method

In the Improper Fraction Method,

- Change the two mixed numbers to improper fractions.
- Multiply the improper fractions.
- Change the product back to a mixed number.

Example: Multiply $\left(2\frac{3}{7}\right) \cdot \left(4\frac{2}{5}\right)$

$$\left(2\frac{3}{7}\right) \cdot \left(4\frac{2}{5}\right)$$

$$= \frac{17}{7} \cdot \frac{22}{5}$$

$$= \frac{374}{35}$$

$$= \mathbf{10\frac{24}{35}}$$

Pre-Algebra
Dividing Mixed Numbers

Improper Fraction Method

To divide mixed numbers, it may be best to use the **Improper Fraction Method**.

Example: Divide $(12\frac{4}{7}) \div (3\frac{1}{14})$

Step 1: Write the starting problem.

$$(12\frac{4}{7}) \div (3\frac{1}{14})$$

Step 2: Convert each mixed number to an improper fraction.

$$= \frac{88}{7} \div \frac{43}{14}$$

Step 3: Convert the division to a multiplication. Remember to "flip that guy and multiply."

$$= \frac{88}{7} \cdot \frac{14}{43}$$

Step 4: Simplify the multiplication if possible.

$$= \frac{88}{1} \cdot \frac{2}{43}$$

Step 5: Multiply both values in the numerator and in the denominator.

$$= \frac{176}{43}$$

Step 6: Divide the numerator by the denominator to obtain the solution to the problem.

$$= \mathbf{4\frac{4}{43}}$$

Step 7: Simplify your answer further, if possible.

No further simplification is possible in the example.

Step 8: If you have a calculator, check your work by converting each fraction to a decimal and seeing of the product generates the solution you calculated.

Check:

$$\left(12\frac{4}{7}\right) \div \left(3\frac{1}{14}\right) \ ? \ \mathbf{4\frac{4}{43}}$$

$$12.57 \div 3.07 \ ? \ 4.09$$

$$4.09 = 4.09$$

✓

The result does not need to be exact, but should be very close. The result in the example is exact to 2 decimals.

Note: If you do not have a calculator, carefully check each step to make sure both your logic and your arithmetic are correct.

Pre-Algebra
Decimal Calculations

Adding and Subtracting Decimals

- Fill in zeroes so that both numbers have the same number of digits after the decimal point.
- Line up the numbers in vertical form.
- Add or subtract.
- Keep the same number of decimals in the result that you have in your vertical form.

Example: $14.02 + 37.1$

$$\begin{array}{r} 14.02 \\ +\ 37.10 \\ \hline \mathbf{51.12} \end{array}$$

Multiplying Decimals

- Line up the numbers in vertical form.
- Multiply as you would if the numbers did not have decimals.
- The result (product) will have a number of decimals equal to the sum of the numbers of decimals in the numbers being multiplied. For example, if a number with 3 decimals is multiplied by a number with 2 decimals, the result will have 5 decimals.

Example: $14.02 \cdot 37.1$

$$\begin{array}{r} 14.02 \\ \cdot\ \ 37.1 \\ \hline 1\,402 \\ 9814 \\ 4206 \\ \hline \mathbf{520.142} \end{array}$$

Dividing Decimals

- Change the original problem. Move the decimal in the divisor to the right until the divisor becomes a whole number. Move the decimal to the right in the dividend the same number of decimals.
- Line up the numbers in long division form.
- The decimal in the quotient will be in the same location as it is in the dividend. Place it there.
- Divide without regard to the decimal.
- Check to see if your answer makes sense. Multiply the quotient and the divisor to see their product is equal to the dividend.

Example: $69.615 \div 2.1$

Rewrite: $696.15 \div 21$

$$\begin{array}{r} 33.15 \\ 21 \overline{\smash{)}\,696.15} \\ \underline{63} \\ 66\,15 \\ \underline{63} \\ 3\,15 \\ \underline{2\,1} \\ 105 \\ \underline{105} \\ 0 \end{array}$$

Pre-Algebra
Comparing Numbers

Comparing Decimals

In comparing decimal numbers, it is useful to add zeroes after the decimal point so that all numbers have the same number of zeroes. To do this,

- Find the number that has the most digits after the decimal point.
- Fill zeroes in the other numbers so they also have that number of digits after the decimal. This will make the problem much easier on your eyes.
- Identify the larger number (or, the order of the numbers if there are more than two).

Example – Trick question: What number is bigger, four-point-eight or four-point-ten?

Solution: Write the numbers: $4.8 \text{ and } 4.10$

Fill in zeroes: $4.80 \text{ and } 4.10$

Compare the numbers: 4.80 is larger

Answer: Four-point-eight is larger than four-point-ten.

Putting Fractions in Numerical Order

In comparing fractions, we use a trick:

- Cross multiply the fractions.
- Put the product of each multiplication on the side of the equal sign as the numerator used in the multiplication.
- Compare the resulting integers.
- The order of the numbers in the original problem is the same as the order of the integers in the cross-multiplication.

Example: Which is larger … $\frac{5}{7}$ or $\frac{3}{4}$?

Solution: Cross Multiply: $5 \cdot 4 \ \ vs. \ \ 3 \cdot 7$

Simplify: $20 < 21$

Because the larger number is on the right, the number on the right in the original problem is also the larger value. Therefore, $\frac{3}{4}$ is the larger fraction.

Alternative: If you have a calculator, you can convert each fraction to a decimal and put the decimals in order.

Pre-Algebra
Rounding Numbers

In rounding a number to the $\mathbf{n^{th}}$ position, look at the numbers to the right of the $\mathbf{n^{th}}$ position. Then,

Rule 1: If the next number to the right is **4 or lower, round down**. That is, replace all of the numbers to the right of the $\mathbf{n^{th}}$ position with zeroes. This is also called **truncation**.

Rule 2: If the next number to the right is **5 and there are no other non-zero numbers to the right, special rules for rounding 5's apply** (see below).

Rule 3: If the next number to the right is **5 followed by any non-zero number anywhere to the right, round up.** That is, add one to the number in the $\mathbf{n^{th}}$ position and replace all of the numbers to the right of the $\mathbf{n^{th}}$ position with zeroes.

Rule 4: If the next number to the right is **6 or higher, round up**.

Special Rules for Rounding 5's

There are two different (and conflicting) rules for rounding 5's. You should make sure that you know which one your teacher requires you to use.

In rounding a number to the n^{th} position, when there are no non-zero digits to the right:

Round Up Rule: Round up whenever there is a 5 in the next position to the right. This rule is often taught in high school to make things easier for the student. The problem with this approach is that if a number of roundings are performed, there is an upward bias in the results.

Round to the Even Digit Rule. Round the number in the $\mathbf{n^{th}}$ position either up or down, using whichever result produces an even number in t he $\mathbf{n^{th}}$ position. This rule is often used in college and business because it does not result in a bias when multiple roundings are performed.

Examples: (note: the number in the $(n+1)^{th}$ place is highlighted in orange)

Round	To what place?	Result	Rule Used
2.843 67	.001	2.844	Round Up – Rule 4
325	10	330 or 320	Rule 3 – depends on "5" Rule
3.141 592 653	.000 000 1	3.141 592 7	Round Up – Rule 2
9,214,387	millions	9,000,000	Round Down – Rule 1
643.915 425 89	millionths	643.915 426	Round Down – Rule 1
2,459.1	tens	2,460	Round Up – Rule 4
42.625 12	hundredths	42.63	Round Up – Rule 2

Pre-Algebra
Conversions of Percents to Decimals and Fractions

The word cent comes from the Latin *centum*, meaning 100. So,

- A penny is a cent.
- A century is 100 years.
- A Roman centurion was in charge of 100 soldiers.

Percent means "per 100" or "out of 100."

Conversion To and From Decimals

A percent is converted to a decimal by moving the decimal point two places to the left.

$$17\% = .17 \qquad\qquad 64\% = .64$$

A decimal is converted to a percent by moving the decimal point two places to the right.

$$.625 = 62.5\% \qquad\qquad .87 = 87\%$$

Conversion To and From Fractions

A percent is converted to a fraction by dividing the percent by 100 and reducing the fraction to lowest terns,

$$25\% = \frac{25}{100} = \frac{1}{4} \qquad\qquad 36\% = \frac{36}{100} = \frac{9}{25}$$

A fraction is converted to a percent by multiplying by 100.

$$\frac{3}{4}\text{:}\ \ \frac{3}{4} \cdot 100\% = \frac{300}{4}\% = 75\% \qquad\qquad \frac{1}{5}\text{:}\ \ \frac{1}{5} \cdot 100\% = \frac{100}{5}\% = 20\%$$

More Examples:

Percent	Decimal	Fraction
0%	0	$\frac{0}{100} = 0$
15%	.15	$\frac{15}{100} = \frac{3}{20}$
44%	.44	$\frac{44}{100} = \frac{11}{20}$
50%	.5	$\frac{50}{100} = \frac{1}{2}$

Percent	Decimal	Fraction
65%	.65	$\frac{65}{100} = \frac{13}{20}$
80%	.8	$\frac{80}{100} = \frac{4}{5}$
95%	.95	$\frac{95}{100} = \frac{19}{20}$
100%	1	$\frac{100}{100} = 1$

Pre-Algebra
Decimal Conversions

1/2	.50	50%	1/8	.125	12.5%
			2/8	.250	25.0%
1/3	.333	33.3%	3/8	.375	37.5%
2/3	.667	66.7%	4/8	.500	50.0%
			5/8	.625	62.5%
1/4	.25	25%	6/8	.750	75.0%
2/4	.50	50%	7/8	.875	87.5%
3/4	.75	75%			
			1/9	.111	11.1%
1/5	.20	20%	2/9	.222	22.2%
2/5	.40	40%	3/9	.333	33.3%
3/5	.60	60%	4/9	.444	44.4%
4/5	.80	80%	5/9	.556	55.6%
			6/9	.667	66.7%
1/6	.167	16.7%	7/9	.778	77.8%
2/6	.333	33.3%	8/9	.889	88.9%
3/6	.500	50.0%			
4/6	.667	66.7%	1/10	.1	10%
5/6	.833	83.3%	2/10	.2	20%
			3/10	.3	30%
1/7	.142857	14.2857%	4/10	.4	40%
2/7	.285714	28.5714%	5/10	.5	50%
3/7	.428571	42.8571%	6/10	.6	60%
4/7	.571429	57.1429%	7/10	.7	70%
5/7	.714286	71.4286%	8/10	.8	80%
6/7	.857143	85.7143%	9/10	.9	90%

1/11	.09091	9.091%	1/16	.06250	6.250%
1/12	.08333	8.333%	1/17	.05882	5.882%
1/13	.07692	7.692%	1/18	.05556	5.556%
1/14	.07143	7.143%	1/19	.05263	5.263%
1/15	.06667	6.667%	1/20	.05000	5.000%

Pre-Algebra
Applying a Percent Increase

It is common in mathematics to work with percent increases. An example of an everyday application of this is the sales tax you pay in the local store. Sales tax is expressed in the form of a percent increase.

Applying a Percent Increase

There are two methods for working with percent increases. Use the one you like best.

Method 1:

- Start with the amount before increase (i.e., the original amount).
- Calculate the amount of the increase.
- Add the original amount and the amount of the increase to obtain the final amount.

$$\binom{increase}{amount} = \binom{original}{amount} \cdot \binom{percent}{increase}$$

$$\binom{final}{amount} = \binom{original}{amount} + \binom{increase}{amount}$$

An advantage of this approach is that you calculate the amount of the increase. Sometimes, this is an important value to know.

Example: What do you get when you increase 150 by 10%?

$$Increase\ Amount = 10\% \cdot 150 = 15$$

$$Final\ Amount = 150 + 15 = 165$$

Method 2:

- Add the percent increase to 100%.
- Multiply the original amount by this new percentage to obtain the final amount.

$$\binom{total}{percent} = 100\% + \binom{percent}{increase}$$

$$\binom{final}{amount} = \binom{original}{amount} \cdot \binom{total}{percent}$$

This approach may be easier and has extensive business applications.

Example: What do you get when you increase 150 by 10%?

$$Total\ Percent = 100\% + 10\% = 110\% = 1.1$$

$$Final\ Amount = 150 \cdot 1.1 = 165$$

Pre-Algebra
Applying a Percent Decrease

It is common in mathematics to work with percent decreases. In a store you may see a sign that says "Sale – 40% off." In such a case, you may want to calculate the sale price.

Applying a Percent Decrease

There are two methods for working with percent decreases. Use the one you like best.

Method 1:

- Start with the amount before decrease (i.e., the original amount).
- Calculate the amount of the decrease.
- Subtract the amount of the decrease from the original amount to obtain the final amount.

$$\binom{decrease}{amount} = \binom{original}{amount} \cdot \binom{percent}{decrease}$$

$$\binom{final}{amount} = \binom{original}{amount} - \binom{decrease}{amount}$$

An advantage of this approach is that you calculate the amount of the decrease. Sometimes, this is an important value to know (e.g., how much money did you save?).

Example: What do you get when you decrease 150 by 40%?

$$Decrease\ Amount = 40\% \cdot 150 = 60$$

$$Final\ Amount = 150 - 60 = 90$$

Method 2:

- Subtract the percent increase from 100%.
- Multiply the original amount by this new percentage to obtain the final amount.

$$\binom{total}{percent} = 100\% - \binom{percent}{decrease}$$

$$\binom{final}{amount} = \binom{original}{amount} \cdot \binom{total}{percent}$$

This approach may be easier and has the same form as the formula for percent increase. It also has extensive business applications.

Example: What do you get when you decrease 150 by 40%?

$$Total\ Percent = 100\% - 40\% = 60\% = 0.6$$

$$Final\ Amount = 150 \cdot 0.6 = 90$$

Pre-Algebra
Calculating Percent Increases and Decreases

Many times, you have the original amount and the final amount after either an increase or decrease in value. You may want to calculate the percent of that increase or decrease.

Percent Increase

Given a starting amount and a final amount,

$$\begin{pmatrix}increase\\amount\end{pmatrix} = \begin{pmatrix}final\\amount\end{pmatrix} - \begin{pmatrix}original\\amount\end{pmatrix}$$

$$\begin{pmatrix}percent\\increase\end{pmatrix} = \begin{pmatrix}amount\ of\\increase\end{pmatrix} \div \begin{pmatrix}original\\amount\end{pmatrix}$$

Example: A stock increases in value from \$80 to \$96; what percent has it increased?

$$increase\ amount = \$96 - \$80 = \$16$$

$$percent\ increase = \$16 \div \$80 = .20 = 20\%$$

Percent Decrease

Given a starting amount and a final amount,

$$\begin{pmatrix}decrease\\amount\end{pmatrix} = \begin{pmatrix}original\\amount\end{pmatrix} - \begin{pmatrix}final\\amount\end{pmatrix}$$

$$\begin{pmatrix}percent\\decrease\end{pmatrix} = \begin{pmatrix}amount\ of\\decrease\end{pmatrix} \div \begin{pmatrix}original\\amount\end{pmatrix}$$

Example: A stock decreases in value from \$80 to \$68; what percent has it increased?

$$decrease\ amount = \$80 - \$68 = \$12$$

$$percent\ decrease = \$12 \div \$80 = .15 = 15\%$$

Notice the following:

- You calculate both an increase and a decrease as the difference between the original and final amounts.
- The percent change is always calculated as the *amount of the change* divided by the *original amount*.

Pre-Algebra
Pie Charts

A Pie Chart is a circular chart that use percentages to show how parts of a whole compare to the whole and to each other. A number of tools are readily available to make pie charts. The ones below were constructed using Microsoft Excel 2007.

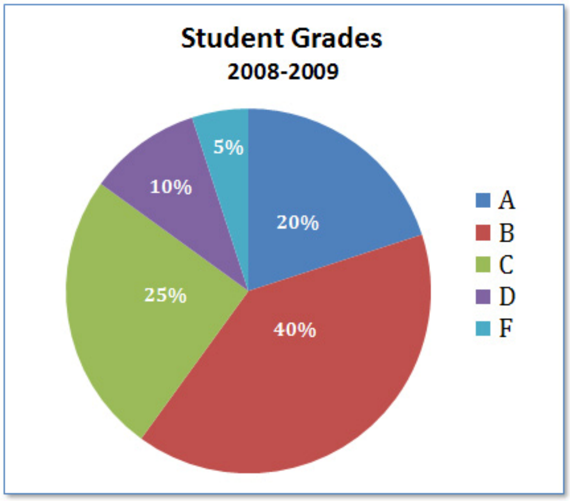
Elements of a Pie Chart

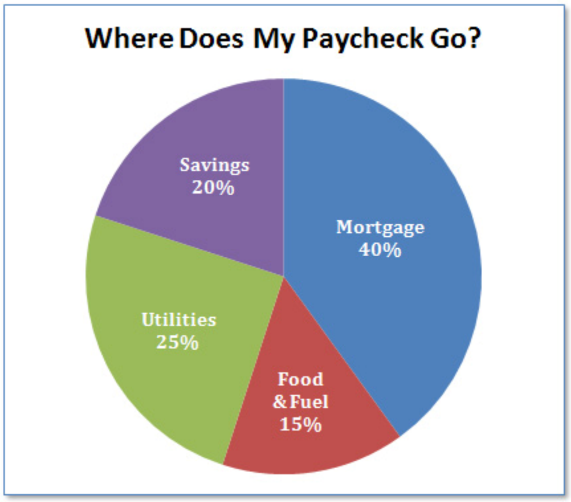
A pie chart should contain the following items:

- Title – A short title should be placed over the chart to describe the information in it. There may be both primary and secondary titles; the secondary titles are optional.
- Body – the body of the chart is a circle with radii that divide the interior of the circle into sectors. Each sector represents a portion of the circle in the same proportion that the item represented by the sector relates to the whole represented by the circle.
- Legend – A legend identifies each sector and tells what it represents. Legends are optional.
- Labels – Labels may be placed in each sector to indicate the percentage it contains or to describe the item it represents.

The sum of the percentages in a pie chart must always add up to 100%.

Sample Pie Charts





The chart on the left uses both labels and a legend to provide information, whereas the chart on the right provides all of its information using labels alone. With pie charts, the key is to provide information; you have a lot of options to be creative as long as you achieve that goal.

Pre-Algebra
Estimating Square Roots

Square Roots of Perfect Squares

The square root of a perfect square can be read right off the table to the right. For example,

$$\sqrt{36} = 6 \quad \textit{because} \quad 6^2 = 36$$

It is worthwhile to memorize the perfect squares in this table. They occur very frequently in math from 7th grade and up.

Square Roots of Other Numbers

Square roots of numbers other than perfect squares can be estimated with a process called interpolation. **To calculate $\sqrt{n}$**:

- Find where n fits between perfect squares in the right hand column of the table to the right.
- Determine the corresponding square roots of the perfect squares above and below n.
- Interpolate between the two square roots in the previous step based on where n lies between the perfect squares.

Example:

Estimate $\sqrt{127}$

127 lies between 121 and 144 in the table to the right.

Line up the three square roots and perfect squares in a table:

		Square Root	Perfect Square		
		11	**121**		
1	$x - 11$	$\boldsymbol{x = \sqrt{127}}$	**127**	6	23
		12	**144**		

Then, solve the proportion: $\dfrac{x-11}{1} = \dfrac{6}{23}$

Cross multiply: $23x - 253 = 6 \quad \Longrightarrow \quad 23x = 259$

Finally, $x = \dfrac{259}{23} = 11.26 \quad \Longrightarrow \quad \boldsymbol{\sqrt{127} = 11.26}$ by interpolation

Using a calculator, the actual value, to 2 decimals, is: $\sqrt{127} = 11.27$ by calculator

Table of Squares

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

$9^2 = 81$

$10^2 = 100$

$11^2 = 121$

$12^2 = 144$

$13^2 = 169$

$14^2 = 196$

$15^2 = 225$

$16^2 = 256$

$17^2 = 289$

$18^2 = 324$

$19^2 = 361$

$20^2 = 400$

Pre-Algebra
Roots of Large Numbers

There are times when it is useful to be able to estimate a rot of a large number. Usually, it is useful to do so to check an answer you have calculated on a calculator.

Square Roots

To estimate the square root of a large number:

- Rewrite the number with its digits in pairs from the right.
- Take the square root of the leftmost digit or pair of digits.
- Count the number of pairs of digits to the right of the leading digit or digits. Add this number of zeroes after the value calculated in the previous step.

Square Root Example:

Take the square root of 162,000

$$\sqrt{162000} \sim \sqrt{16\ 20\ 00}$$

Leftmost Digits | 2 Pairs of Digits on Right

$\sqrt{16} = 4$ | Add 2 zeroes

Estimate is 400

Check: $400^2 = 160{,}000$

Higher Level Roots

To estimate the n-th root of a large number:

- Rewrite the number with its digits in groups of size n from the right.
- Take the n-th root of the leftmost set of digits.
- Count the number of sets of digits to the right of the leading digit or digits. Add this number of zeroes after the value calculated in the previous step.

3rd (Cube) Root Example:

Take the 3rd root of 27,000,000,000

$$\sqrt{27{,}000{,}000{,}000} \sim \sqrt{27\ 000\ 000\ 000}$$

Leftmost Digits | 3 Sets of 3 Digits on Right

$\sqrt[3]{27} = 3$ | Add 3 zeroes

Estimate is 3,000

Check: $3{,}000^3 = 27{,}000{,}000{,}000$

4th Root Example:

Take the 4th root of 62,512,256,013

$$\sqrt{62{,}512{,}256{,}013} \sim \sqrt{625\ 1225\ 6013}$$

Leftmost Digits | 2 Sets of 4 Digits on Right

$\sqrt[4]{625} = 5$ | Add 2 zeroes

Estimate is 500

Check: $500^4 = 62{,}500{,}000{,}000$

Pre-Algebra
Exponent Formulas

Word Description of Property	Math Description of Property	Limitations on Variables	Examples
Product of Powers	$a^p \cdot a^q = a^{(p+q)}$		$x^4 \cdot x^3 = x^7$ $x^5 \cdot x^{-8} = x^{-3}$
Quotient of Powers	$\dfrac{a^p}{a^q} = a^{(p-q)}$	$a \neq 0$	$\dfrac{y^5}{y^2} = y^3$
Power of a Power	$(a^p)^q = a^{(p \cdot q)}$		$(z^4)^3 = z^{12}$ $(x^{-3})^{-5} = x^{15}$
Anything to the zero power is 1	$a^0 = 1$	$a \neq 0$	$91^0 = 1$ $(xyz^3)^0 = 1$, if $x, y, z \neq 0$
Negative powers generate the reciprocal of what a positive power generates	$a^{(-p)} = \dfrac{1}{a^p}$	$a \neq 0$	$x^{(-3)} = \dfrac{1}{x^3}$ $\left(\dfrac{1}{x}\right)^{-5} = x^5$
Power of a product	$(a \cdot b)^p = a^p \cdot b^p$		$(3y)^3 = 27y^3$
Power of a quotient	$\left(\dfrac{a}{b}\right)^p = \dfrac{a^p}{b^p}$	$b \neq 0$	$\left(\dfrac{x}{4}\right)^3 = \dfrac{x^3}{64}$
Converting a root to a power	$\sqrt[n]{a} = a^{(1/n)}$	$n \neq 0$	$\sqrt{x} = x^{1/2}$

Pre-Algebra
Powers of 10

Uses of Powers of 10

Powers of 10 are useful in mathematics and science. In particular, they are used in scientific notation to express very large numbers and very small numbers without using up all the space a bunch of zeroes would take. Numbers with a lot of zeroes are also hard to grasp, whereas powers of 10 are relatively easy to grasp.

Negative Powers of 10

For negative powers of 10, the number of zeroes before the 1, including one zero to the left of the decimal point, is equal to the exponent (disregarding the negative sign).

Zero Power of 10

$10^0 = 1$ *(notice, no zeroes to the left or right of the 1)*

Positive Powers of 10

For positive powers of 10, the number of zeroes after the 1 is equal to the exponent.

Fun Only – Special Cases

There are two special cases for powers of 10 that mathematicians have defined. For very big numbers, mathematicians have defined the googol and the googolplex. These are not to be confused with Google, the internet search engine; they are spelled differently.

They are defined as:

$googol = 10^{100}$ (a 1 followed by 100 zeroes)

$googolplex = 10^{\mathrm{googol}}$ (a 1 followed by googol zeroes)

Maybe you can create your own name for:

$10^{\mathrm{googolplex}}$ (a 1 followed by googolplex zeroes)

Powers of 10

$10^{-6} = 0.000\ 001$

$10^{-5} = 0.000\ 01$

$10^{-4} = 0.000\ 1$

$10^{-3} = 0.001$

$10^{-2} = 0.01$

$10^{-1} = 0.1$

$10^0 = 1$

$10^1 = 10$

$10^2 = 100$

$10^3 = 1{,}000$

$10^4 = 10{,}000$

$10^5 = 100{,}000$

$10^6 = 1{,}000{,}000$

$10^{100} = \mathrm{googol}$

$10^{\mathrm{googol}} = googolplex$

Pre-Algebra
Scientific Notation

Format

A number in scientific notation has two parts:

- A number which is at least 1 and is less than 10 (i.e., it must have only one digit before the decimal point). This number is called the **coefficient**.
- A power of 10 which is multiplied by the first number.

Here are a few **examples** of regular numbers expressed in scientific notation.

$$32 = 3.2\ x\ 10^1 \qquad 0.00034 = 3.4\ x\ 10^{-4} \qquad 1 = 1\ x\ 10^0$$

$$1{,}420{,}000 = 1.42\ x\ 10^6 \qquad 1000 = 1\ x\ 10^3 \qquad -450 = -4.5\ x\ 10^2$$

How many digits? How many zeroes?

There are a couple of simple rules for converting from scientific notation to a regular number or for converting from a regular number to scientific notation:

- If a regular number is less than 1, the exponent of 10 in scientific notation is negative. The number of leading zeroes in the regular number is equal to the absolute value of this exponent. In applying this rule, you must count the zero before the decimal point in the regular number. Examples:

Original Number	Action	Conversion
0.00034	Count 4 zeroes	3.4×10^{-4}
6.234×10^{-8}	Add 8 zeroes before the digits	0.000 000 062 34

- If the number is greater than 1, the number of digits after the first one in the regular number is equal to the exponent of 10 in the scientific notation.

Original Number	Action	Conversion
4,800,000	Count 6 digits after the “4”	4.8×10^{6}
9.6×10^{3}	Add 3 digits after the “9”	9,600

- As a general rule, multiplying by powers of 10 moves the decimal point one place for each power of 10.
  - Multiplying by positive powers of 10 moves the decimal to the right.
  - Multiplying by negative powers of 10 moves the decimal to the left.

Pre-Algebra

Adding and Subtracting with Scientific Notation

When adding or subtracting numbers in scientific notation:

- Adjust the numbers so they have the same power of 10. This works best if you adjust the representation of the smaller number so that it has the same power of 10 as the larger number. To do this:
  - Call the difference between the exponents of 10 in the two numbers *"n"*.
  - Raise the power of 10 of the smaller number by *"n"*, and
  - Move the decimal point of the coefficient of the smaller number *"n"* places to the left.
- Add the coefficients, keeping the power of 10 unchanged.
- If the result is not in scientific notation, adjust it so that it is.
  - If the coefficient is at least 1 and less than 10, the answer is in the correct form.
  - If the coefficient is 10 or greater, increase the exponent of 10 by 1 and move the decimal point of the coefficient one space to the left.
  - If the coefficient is less than 1, decrease the exponent of 10 by 1 and move the decimal point of the coefficient one space to the right.

Examples:

$$\begin{array}{rcr} 3.2\ x\ 10^{3} & \longrightarrow & 0.32\ x\ 10^{4} \\ +9.9\ x\ 10^{4} & \longrightarrow & \underline{+9.90\ x\ 10^{4}} \\ & & 10.22\ x\ 10^{4} \\ & & =\ 1.022\ x\ 10^{5} \end{array}$$

Explanation: A conversion of the smaller number is required prior to adding because the exponents of the two numbers are different. After adding, the result is no longer in scientific notation, so an extra step is needed to convert it into the appropriate format.

$$\begin{array}{rcr} 6.1\ x\ 10^{-2} & \longrightarrow & 6.1\ x\ 10^{-2} \\ +2.3\ x\ 10^{-2} & \longrightarrow & \underline{+2.3\ x\ 10^{-2}} \\ & & 8.4\ x\ 10^{-2} \end{array}$$

Explanation: No conversion is necessary because the exponents of the two numbers are the same. After adding, the result is in scientific notation, so no additional steps are required.

$$\begin{array}{rcr} 1.2\ x\ 10^{-8} & \longrightarrow & 1.20\ x\ 10^{-8} \\ -4.5\ x\ 10^{-9} & \longrightarrow & \underline{-0.45\ x\ 10^{-8}} \\ & & 0.75\ x\ 10^{-8} \\ & & =\ 7.5\ x\ 10^{-9} \end{array}$$

Explanation: A conversion of the smaller number is required prior to subtracting because the exponents of the two numbers are different. After subtracting, the result is no longer in scientific notation, so an extra step is needed to convert it into the appropriate format.

Pre-Algebra

Multiplying and Dividing with Scientific Notation

When multiplying or dividing numbers in scientific notation:

- Multiply or divide the coefficients.
- Multiply or divide the powers of 10. Remember that this means adding or subtracting the exponents while keeping the base of 10 unchanged.
    - If you are multiplying, add the exponents of 10.
    - If you are dividing, subtract the exponents of 10.
- If the result is not in scientific notation, adjust it so that it is.
    - If the coefficient is at least 1 and less than 10, the answer is in the correct form.
    - If the coefficient is 10 or greater, increase the exponent of 10 by 1 and move the decimal point of the coefficient one space to the left.
    - If the coefficient is less than 1, decrease the exponent of 10 by 1 and move the decimal point of the coefficient one space to the right.

Examples:

$$\begin{array}{r} 4\ x\ 10^{6} \\ \cdot\, 5\ x\ 10^{-4} \\ \hline 20\ x\ 10^{2} \\ = 2.0\ x\ 10^{3} \end{array}$$

Explanation: The coefficients are multiplied and the exponents are added. After multiplying, the result is no longer in scientific notation, so an extra step is needed to convert it into the appropriate format.

$$\begin{array}{r} 1.2\ x\ 10^{-2} \\ \cdot\ 2.0\ x\ 10^{-6} \\ \hline 2.4\ x\ 10^{-8} \end{array}$$

Explanation: The coefficients are multiplied and the exponents are added. After multiplying, the result is in scientific notation, so no additional steps are required.

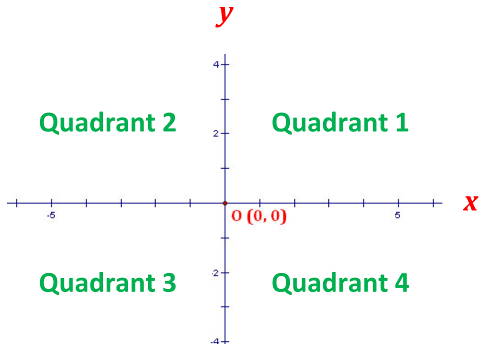
$$\begin{array}{r} 3.3\ x\ 10^{4} \\ \div\ 5.5\ x\ 10^{-2} \\ \hline 0.6\ x\ 10^{6} \\ = 6.0\ x\ 10^{5} \end{array}$$

Explanation: The coefficients are divided and the exponents are subtracted. After dividing, the result is no longer in scientific notation, so an extra step is needed to convert it into the appropriate format.

Pre-Algebra
Graphing with Coordinates

Graphs in two dimensions are very common in algebra and are one of the most common algebra applications in real life.

Coordinates



The plane of points that can be graphed in 2 dimensions is called the Rectangular Coordinate Plane or the Cartesian Coordinate Plane (named after the French mathematician and philosopher René Descartes).

- Two axes are defined (usually called the x- and y-axes).
- Each point on the plane has an x value and a y value, written as: **(*x-value, y-value*)**
- The point (0, 0) is called the origin, and is usually denoted with the letter "O".
- The axes break the plane into 4 quadrants, as shown above. They begin with Quadrant 1 where x and y are both positive and increase numerically in a counter-clockwise fashion.

Plotting Points on the Plane

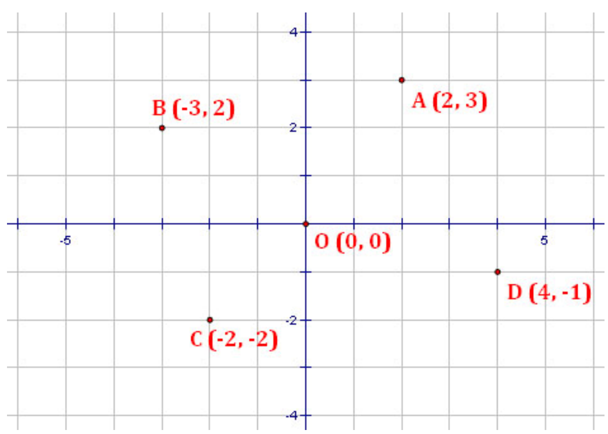
When plotting points,

- the x-value determines how far right (positive) or left (negative) of the origin the point is plotted.
- The y-value determines how far up (positive) or down (negative) from the origin the point is plotted.

Examples:

The following points are plotted in the figure to the right:

A = (2, 3)	in Quadrant 1
B = (-3, 2)	in Quadrant 2
C = (-2, -2)	in Quadrant 3
D = (4, -1)	in Quadrant 4
O = (0, 0)	is not in any quadrant



Pre-Algebra
Changing Words to Mathematical Expressions

Word Problems use words that must be translated into mathematical expressions to be solved. In these problems, certain words are used repeatedly. Here are some of them.

Category of Words	Words	Sample Word Expression	Math Translation
Addition	added to increased by more than plus sum of total of	6 added to 4 21 increased by 6 5 more than 9 The total of 3 and 7	$6 + 4 = 10$ $21 + 6 = 27$ $5 + 9 = 14$ $3 + 7 = 10$
Subtraction	decreased by difference between difference of less less than lower than minus reduced by subtracted from	13 decreased by 6 Difference of 42 and 24 6 less than 9 12 lower than 33 11 reduced by 3	$13 - 6 = 7$ $42 - 24 = 18$ $9 - 6 = 3$ $33 - 12 = 21$ $11 - 3 = 8$
Multiplication	double multiplied by of product of times triple twice	Double 12 Product of 3 and 6 14 times 3 Triple 7	$2 \cdot 12 = 24$ $3 \cdot 6 = 18$ $14 \cdot 3 = 42$ $3 \cdot 7 = 21$
Division	divided among divided by half one-third out of per quotient of ratio of	12 divided by 6 Half of 24 One third of 18 Quotient of 32 and 16 Ratio of 24 and 6	$12 \div 6 = 2$ $24 \div 2 = 12$ $18 \div 3 = 6$ $32 \div 16 = 2$ $24 \div 6 = 4$
Equals	equals generates gives is, are, other forms of "to be" provides yields	6 and 4 generates x 4 lower than x gives 6 x is triple 4 12 out of x is $\frac{1}{2}$	$6 + 4 = x$ $x - 4 = 6$ $x = 3 \cdot 4$ $\frac{12}{x} = \frac{1}{2}$

Pre-Algebra
Solving One-Step Equations

The main thrust in a one-step equation is to isolate the variable on one side of the equation, and have all the "numbers" on the other side. This is accomplished by "un-doing" whatever operation in the equation is interacting with the variable. By definition, a one-step equation requires only one step to develop a solution.

Operations are "un-done" by applying the opposite (or, inverse) operation on the equation. Inverse operations are listed in the following table:

Operation	Inverse Operation
Addition	Subtraction
Subtraction	Addition
Multiplication	Division
Division	Multiplication
Exponent	Logarithm
Logarithm	Exponent

Examples:

Example 1:

Solve: $x + 4 = 9$

Subtract 4: $-4 \quad -4$

Result: $x = 5$

Example 2:

Solve: $x - 3 = -6$

Add 3: $+3 \quad +3$

Result: $x = -3$

Example 3:

Solve: $3x = -15$

Divide by 3: $\frac{3x}{3} = \frac{-15}{3}$

Result: $x = -5$

Example 4:

Solve: $\frac{1}{6}x = 4$

Multiply by 6: $\cdot 6 \quad \cdot 6$

Result: $x = 24$

Pre-Algebra
Solving Multi-Step Equations

Reverse PEMDAS

One systematic way to approach multi-step equations is Reverse PEMDAS. PEMDAS describes the order of operations used to evaluate an expression. Solving an equation is the opposite of evaluating it, so reversing the PEMDAS order of operations seems appropriate.

The **guiding principles** in the process are:

- Each step works toward isolating the variable for which you are trying to solve.
- Each step "un-does" an operation in **Reverse PEMDAS** order:

Subtraction	←Inverses→	**A**ddition
Division	←Inverses→	**M**ultiplication
Exponents	←Inverses→	Logarithms
Parentheses	←Inverses→	Remove Parentheses (and repeat process)

Note: Logarithms are the inverse operator to exponents. This topic is typically covered in the second year of Algebra.

The list above shows inverse operation relationships. In order to undo an operation, you perform its inverse operation. For example, to undo addition, you subtract; to undo division, you multiply. Here are a couple of examples:

Example 1

Solve:	$3x - 4$	$= 14$
Step 1: Add 4	$+4$	$+4$
Result:	$3x$	$= 18$
Step 2: Divide by 3	$\div 3$	$\div 3$
Result:	x	$= 6$

Notice that we add and subtract before we multiply and divide. **Reverse PEMDAS.**

Example 2

Solve:	$2 \cdot (2x + 5) - 3$	$= -5$
Step 1: Add 3	$+3$	$+3$
Result:	$2 \cdot (2x + 5)$	$= -2$
Step 2: Divide by 2	$\div 2$	$\div 2$
Result:	$(2x + 5)$	$= -1$
Step 3: Remove parentheses		
Result:	$2x + 5$	$= -1$
Step 4: Subtract 5	-5	-5
Result:	$2x$	$= -6$
Step 5: Divide by 2	$\div 2$	$\div 2$
Result:	x	$= -3$

With this approach, you will be able to solve almost any multi-step equation. As you get better at it, you will be able to use some shortcuts to solve the problem faster. Since speed is important in mathematics, learning a few tips and tricks with regard to solving equations is likely to be worth your time.

Pre-Algebra
Tips and Tricks in Solving Multi-Step Equations

Fractional Coefficients

Fractions present a stumbling block to many students in solving multi-step equations. When stumbling blocks occur, it is a good time to develop a trick to help with the process. The trick shown below involves using the reciprocal of a fractional coefficient as a multiplier in the solution process. (Remember that a coefficient is a number that is multiplied by a variable.)

Example 1

Solve:	$\frac{2}{3}x = 8$
Multiply by $\frac{3}{2}$:	$\cdot\frac{3}{2} \quad \cdot\frac{3}{2}$
Result:	$x = \frac{3}{2} \cdot 8 = \frac{24}{2} = 12$

Explanation: Since $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$, when we multiply them, we get 1, and $1 \cdot x = x$. Using this approach, we can avoid dividing by a fraction, which is more difficult.

Example 2

Solve:	$-\frac{1}{4}x = -2$
Multiply by -4:	$\cdot(-4) \quad \cdot(-4)$
Result:	$x = (-2) \cdot (-4) = 8$

Explanation: -4 is the reciprocal of $-\frac{1}{4}$, so when we multiply them, we get 1. Notice the use of parentheses around the negative number to make it clear we are multiplying and not subtracting.

Another Approach to Parentheses

In the Reverse PEMDAS method, parentheses are handled after all other operations. Sometimes, it is easier to operate on the parentheses first. In this way, you may be able to re-state the problem in an easier form before solving it.

Example 3, at right, is another look at the problem in Example 2 on the previous page.

Use whichever approach you find most to your liking. They are both correct.

Example 3

Solve:	$2 \cdot (2x + 5) - 3 = -5$
Step 1: Eliminate parentheses	
Result:	$4x + 10 - 3 = -5$
Step 2: Combine constants	
Result:	$4x + 7 = -5$
Step 3: Subtract 7	$-7 \quad -7$
Result:	$4x = -12$
Step 4: Divide by 4	$\div 4 \quad \div 4$
Result:	$x = -3$

Pre-Algebra
Solving for a Variable

Sometimes in mathematics and, especially, in science, we are presented with a formula and asked to solve for one of its variables. This kind of problem is an application in solving an equation. The key difference from other equations the student has been asked to solve is that there are usually several variables in the expression.

The Trick

The trick is to treat the variables the same way you would treat numbers. Just pretend they are numbers. You can add, subtract, multiply, divide and more, just like you would numbers. Your goal is to isolate the variable you are solving for.

Example 1: Triangle Perimeter

The formula for the perimeter of a triangle is:

$$P = a + b + c$$

Solve this for a.

Here are the steps:

Start:	$P = a + b + c$
Subtract b:	$\quad\quad -b \quad\quad -b$
Result:	$P - b = a + c$
Subtract c:	$\quad -c \quad\quad -c$
Result:	$P - b - c = a$
Switch sides:	$\boldsymbol{a = P - b - c}$

Example 2: Ideal Gas Law

The Ideal Gas Law states that:

$$PV = nRT$$

Solve this for T.

Here are the steps:

Start:	$PV = nRT$
Divide by n:	$\div n \quad \div n$
Result:	$\dfrac{PV}{n} = RT$
Divide by R:	$\div R \quad \div R$
Result:	$\dfrac{PV}{nR} = T$
Switch sides:	$\boldsymbol{T = \dfrac{PV}{nR}}$

The main difficulty in solving problems with multiple variables is keeping track of all the variables. Here are some tips:

- Take your time and be careful.
- Perform the calculations one at a time and in a logical progression.
- Check your work carefully. If you did your work correctly, you should be able to reverse your steps and derive the original formula from the one in your solution.

Pre-Algebra
Inequalities

Inequality Signs

The following signs are used in inequalities;

- $<$ – Less than sign. $a < b$ is read "a is less than b."
- $\leq$ – Less than or equal sign. $a \leq b$ is read "a is less than or equal to b."
- $>$ – Greater than sign. $a > b$ is read "a is greater than b."
- $\geq$ – Greater than or equal sign. $a \geq b$ is read "a is greater than or equal to b."

Relationship to Equations

Inequalities are solved in much the same way as equations. There are a couple of differences you should be aware of:

- When you multiply or divide by a negative number, you must flip the sign. That is,
  " $<$ " becomes " $>$ "
  " $\leq$ " becomes " $\geq$ "
  " $>$ " becomes " $<$ "
  " $\geq$ " becomes " $\leq$ "
- When you switch sides of an inequality you must flip the sign.

Examples:

Example 1:

Solve:	$x + 4 \leq -1$
Subtract 4:	$-4 \quad -4$
Result:	$x \leq -5$

Example 2:

Solve:	$-3x > 9$
Divide by –3:	$\div(-3) \quad \div(-3)$
Flip Sign:	$x < -3$

A Trick – Think about the two sides of the inequality signs. The pointy side is small, and the open side is large. Then,

- The smaller expression is on the pointy (smaller) side of the sign.
- The larger expression is on the open (larger) side of the sign.

Example 3:

Switch sides:	$6 < x$
Flip Sign:	$x > 6$

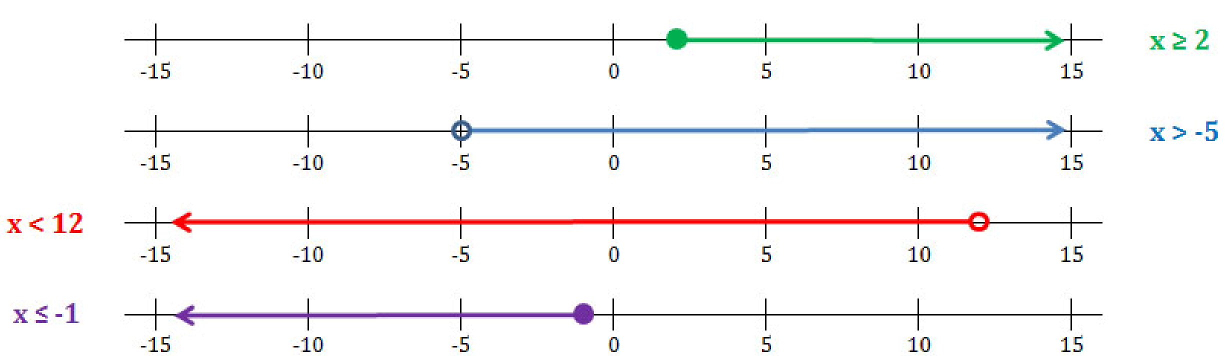
Pre-Algebra
Graphs of Inequalities in One Dimension

Inequalities in one dimension are generally graphed on the number line. Alternatively, if it is clear that the graph is one-dimensional, the graphs can be shown in relation to a number line but not specifically on it (examples of this are on the next page).

One-Dimensional Graph Components

- **The endpoint(s)** – The endpoints for the ray or segment in the graph are shown as either open or closed circles.
  - If the point is included in the solution to the inequality (i.e., if the sign is ≤ or ≥), the circle is closed.
  - If the point is not included in the solution to the inequality (i.e., if the sign is < or >), the circle is open.
- **The arrow** – If all numbers in one direction of the number line are solutions to the inequality, an arrow points in that direction.
  - For < or ≤ signs, the arrow points to the left ($\longleftarrow$).
  - For > or ≥ signs, the arrow points to the right ($\longrightarrow$).
- **The line** – in a simple inequality, a line is drawn from the endpoint to the arrow. If there are two endpoints, a line is drawn from one to the other.

Examples:



Pre-Algebra
Compound Inequalities in One Dimension

Compound inequalities are a set of inequalities that must all be true at the same time. Usually, there are two inequalities, but more than two can also form a compound set. The principles described below easily extend to cases where there are more than two inequalities.

Compound Inequalities with the Word "AND"

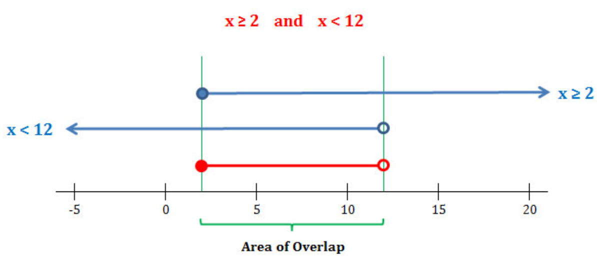
An example of compound inequalities with the word "AND" would be:

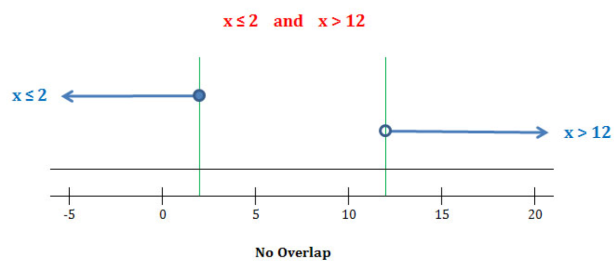
$x < 12 \;\; and \;\; x \geq 2$ **or** $2 \leq x < 12$

(Simple Form) **(Compound Form)**

These are the same conditions, expressed in two different forms.

Graphically, "AND" inequalities exist at points where the graphs of the individual inequalities overlap. This is the "intersection" of the graphs of the individual inequalities. Below are two examples of graphs of compound inequalities using the word "AND."



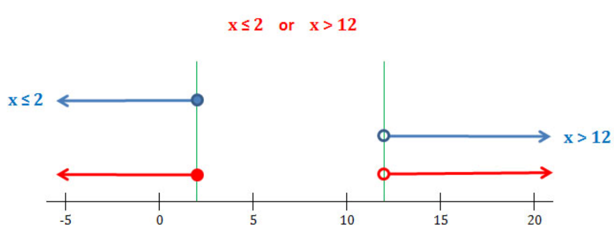


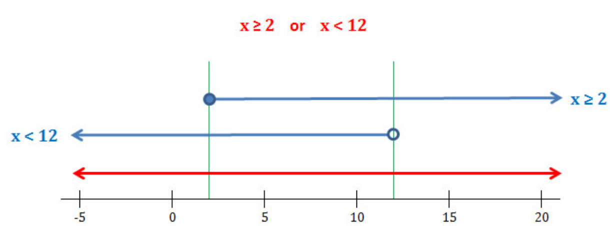
A typical "AND" example: The result is a segment that contains the points that overlap the graphs of the individual inequalities.

"AND" compound inequalities sometimes result in the empty set. This happens when no numbers meet both conditions at the same time.

Compound Inequalities with the Word "OR"

Graphically, "OR" inequalities exist at points where <u>any</u> of the original graphs have points. This is the "union" of the graphs of the individual inequalities. Below are two examples of graphs of compound inequalities using the word "OR."





A typical "OR" example: The result is a pair of rays extending in opposite directions, with a gap in between.

"OR" compound inequalities sometimes result in the set of all numbers. This happens when every number meets at least one of the conditions.

Pre-Algebra
t-Charts

One of the easiest ways to plot a line is to create something called a **t-chart**. To develop a t-chart,

- Draw a large lower case "t" on your page. Also, draw a set of x and y axes on your page.
- On the t-chart, label the top: "x" on the left and "y" on the right.
- Select a set of at least three x-values; place them on the left hand side of the "t." **Tip:** Usually zero is a good choice for x; you may also want to use a negative number and a positive number. However, and three values will work.
- Calculate the corresponding y-values for each x-value.
- Plot the (x, y) ordered pairs on the axes you drew in the first step.
- Draw the line through those points. If the line does not go through all of the points, you have made a mistake; check your work carefully.

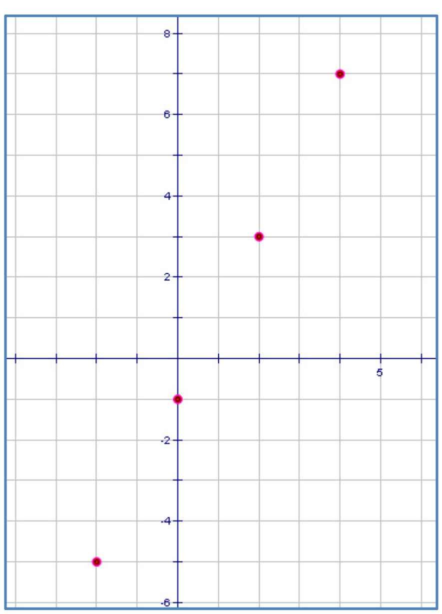
Example:

Draw the graph of the line: $y = 2x - 1$.

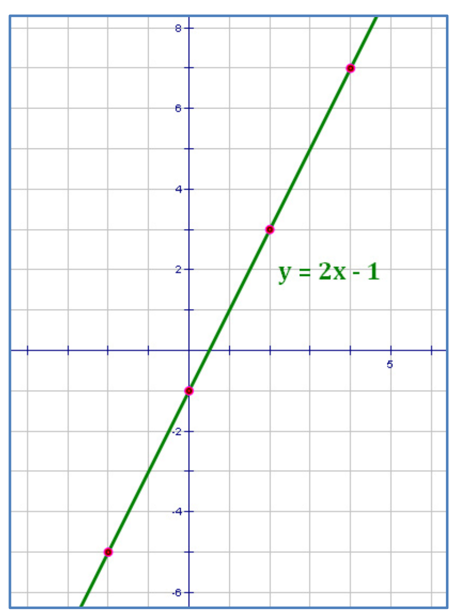
Create a T-chart:

x	y
-2	$y = 2(-2) - 1 = -5$
0	$y = 2(0) - 1 = -1$
2	$y = 2(2) - 1 = 3$
4	$y = 2(4) - 1 = 7$

Plot the Points:



Draw the Line:

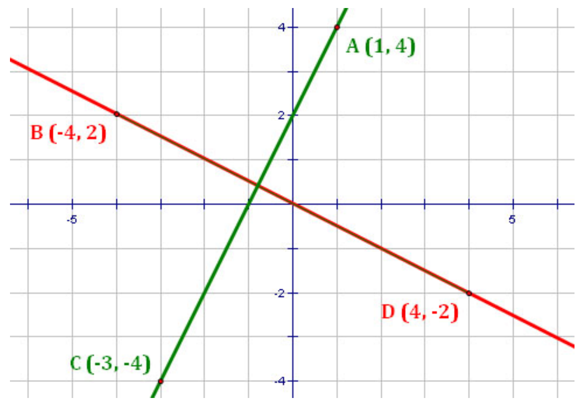


Pre-Algebra
Slope of a Line

The slope of a line tells how fast it rises or falls as it moves from left to right. If the slope is rising, the slope is positive; if it is falling, the slope is negative. The letter "m" is often used as the symbol for slope.

The two most useful ways to calculate the slope of a line are discussed below.

Mathematical Definition of Slope



The definition is based on two points with coordinates (x_1, y_1) and (x_2, y_2). The definition, then, is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Comments:

- You can select any 2 points on the line.
- A table such as the one at right can be helpful for doing your calculations.
- Note that $m = \frac{y_2 - y_1}{x_2 - x_1}$ implies that $m = \frac{y_1 - y_2}{x_1 - x_2}$. So, it does not matter which point you assign as Point 1 and which you assign as Point 2. Therefore, neither does it matter which point is first in the table.
- It is important that once you assign a point as Point 1 and another as Point 2, that you use their coordinates in the proper places in the formula.

	x-value	y-value
Point 2	x_2	y_2
Point 1	x_1	y_1
Difference	$x_2 - x_1$	$y_2 - y_1$

Examples:

For the two lines in the figure above, we get the following:

Green Line	**x-value**	**y-value**
Point A	1	4
Point C	-3	-4
Difference	4	8

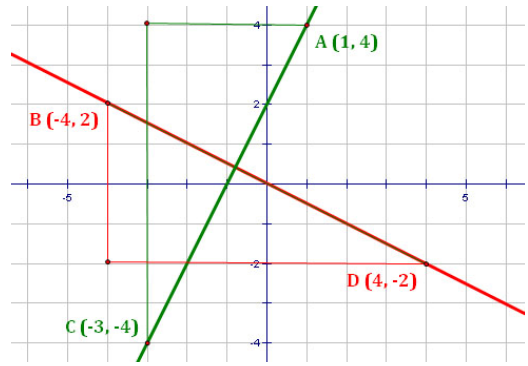
Red Line	**x-value**	**y-value**
Point D	4	-2
Point B	-4	2
Difference	8	-4

Green Line: $m = \frac{8}{4} = 2$

Red Line: $m = \frac{-4}{8} = -\frac{1}{2}$

Pre-Algebra
Slope of a Line (cont'd)

Rise over Run



An equivalent method of calculating slope that is more visual is the "Rise over Run" method. Under this method, it helps to draw vertical and horizontal lines that indicate the horizontal and vertical distances between points on the line.

The slope can then be calculated as follows:

$$m = \frac{rise}{run} = \frac{length\ of\ vertical\ line}{length\ of\ horizontal\ line}$$

The **rise** of a line is how much it **increases (positive)** or **decreases (negative)** between two points. The **run** is how far the line moves to the **right (positive)** or the **left (negative)** between the same two points.

Comments:

- You can select any 2 points on the line.
- It is important to start at the same point in measuring both the rise and the run.
- A good convention is to always start with the point on the left and work your way to the right; that way, the run (i.e., the denominator in the formula) is always positive. The only exception to this is when the run is zero, in which case the slope is undefined.
- If the two points are clearly marked as integers on a graph, the rise and run may actually be counted on the graph. This makes the process much simpler than using the formula for the definition of slope. However, when counting, make sure you get the right sign for the slope of the line, e.g., moving down as the line moves to the right is a negative slope.

Examples:

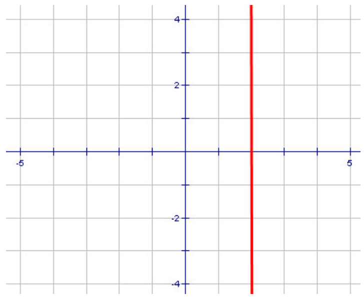
For the two lines in the figure above, we get the following:

Green Line: $m = \dfrac{rise\ from (-4)\ to\ 4}{run\ from\ (-3) to\ 1} = \dfrac{8}{4} = 2$

Red Line: $m = \dfrac{fall\ from\ 2\ to\ (-2)}{run\ from\ (-4)\ to\ 4} = \dfrac{-4}{8} = -\dfrac{1}{2}$

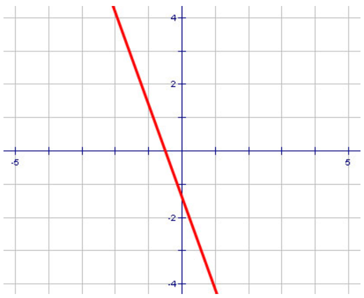
Notice how similar the calculations in the examples are under the two methods of calculating slopes.

Pre-Algebra
Slopes of Various Lines



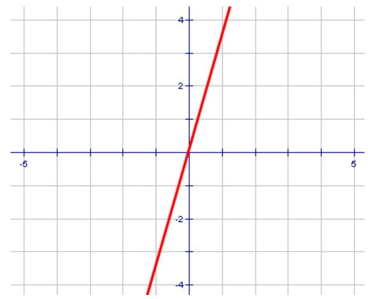
$m = \ undefined$

line is vertical



$m = \ -2\frac{4}{5}\ (big\ negative)$

line is steep and going down



$m = 3\frac{1}{2}\ (big\ positive)$

line is steep and going up

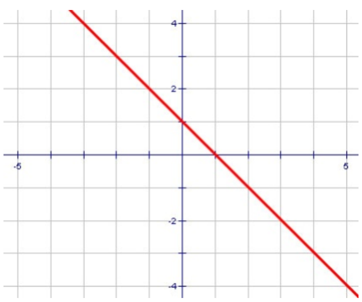
When you look at a line, you should notice the following about its slope:

- *Whether it is 0, positive, negative or undefined.*
- *If positive or negative, whether it is less than 1, about 1, or greater than 1.*

The purpose of the graphs on this page is to help you get a feel for these things.

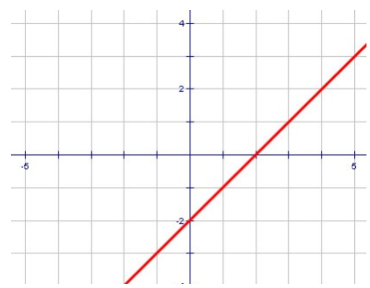
This can help you check:

- Given a slope, whether you drew the line correctly, or
- Given a line, whether you calculated the slope correctly.



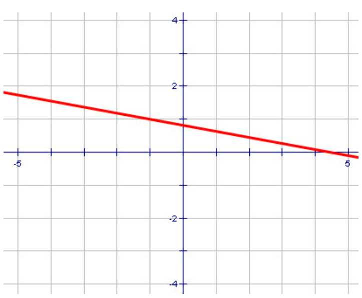
$m = \ -1$

line goes down at a 45^{o} angle



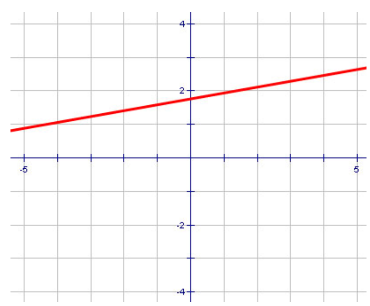
$m = 1$

line goes up at a 45^{o} angle



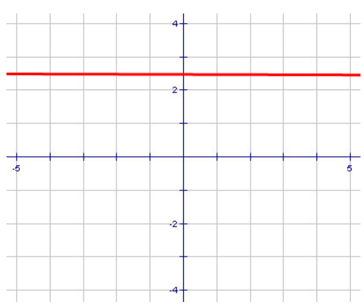
$m = \ -\frac{3}{17}\ (small\ negative)$

line is shallow and going down



$m = \frac{2}{11}\ (small\ positive)$

line is shallow and going up



$m = \ 0$

line is horizontal

Pre-Algebra
Various Forms of a Line

There are three forms of a linear equation which are most useful to the Algebra student, each of which can be converted into the other two through algebraic manipulation. The ability to move between forms is a very useful skill in Algebra, and should be practiced by the student.

Standard Form

The Standard Form of a linear equation is:

$$\boldsymbol{Ax + By = C}$$

where A, B, and C are real numbers and A and B are not both zero. Usually in this form, the convention is for A to be positive.

Standard Form Examples

$$3x + 2y = 6$$

$$2x - 7y = 14$$

Why, you might ask, is this "Standard Form?" One reason is that this form is easily extended to additional variables, whereas other forms are not. For example, in four variables, the Standard Form would be: $\boldsymbol{Ax + By + Cz + Dw = E}$. Another reason is that this form easily lends itself to analysis with matrices, which can be very useful in solving systems of equations.

Slope-Intercept Form

The Slope-Intercept Form of a linear equation is the one most familiar to many students. It is:

$$\boldsymbol{y = mx + b}$$

where m is the slope and b is the y-intercept of the line (i.e., the value at which the line crosses the y-axis in a graph). m and b must also be real numbers.

Slope-Intercept Examples

$$y = -3x + 6$$

$$y = \frac{3}{4}x - 14$$

Point-Slope Form

The Point-Slope Form of a linear equation is the one used least by the student, but it can be very useful in certain circumstances. In particular, as you might expect, it is useful if the student is asked for the equation of a line and is given the line's slope and the coordinates of a point on the line. The form of the equation is:

$$\boldsymbol{(y - y_1) = m(x - x_1)}$$

where m is the slope and (x_1, y_1) is any point on the line. One strength of this form is that **equations formed using different points on the same line will be equivalent.**

Point-Slope Examples

$$(y - 3) = 2\ (x + 4)$$

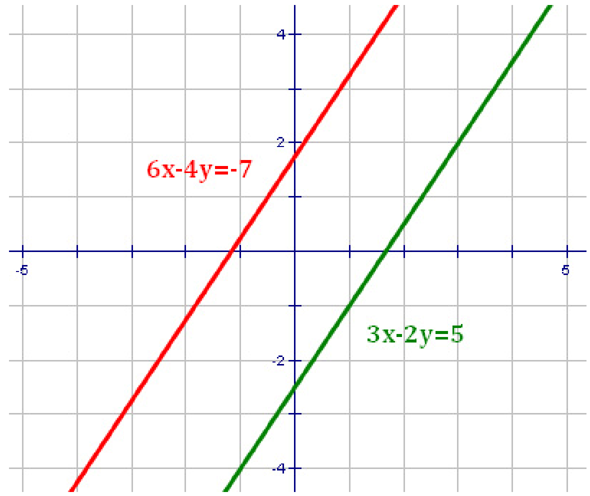
$$(y + 7) = 5\ \left(x - \frac{2}{3}\right)$$

Pre-Algebra

Slopes of Parallel and Perpendicular Lines

Parallel Lines

Two lines are parallel if their slopes are equal.



- In $y = mx + b$ form, if the values of m are the same.

  Example: $y = 2x - 3$ and
  $y = 2x + 1$

- In Standard Form, if the coefficients of x and y are proportional between the equations.

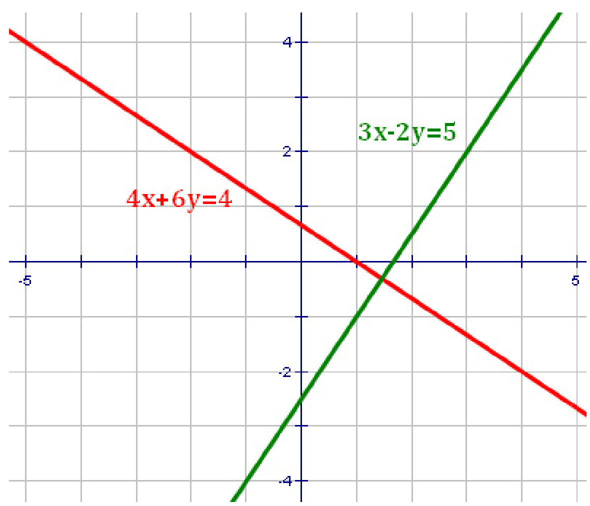
  Example: $3x - 2y = 5$ and
  $6x - 4y = -7$

- Also, if the lines are both vertical (i.e., their slopes are undefined).

  Example: $x = -3$ and
  $x = 2$

Perpendicular Lines

Two lines are perpendicular if the product of their slopes is -1. That is, if the slopes have different signs and are multiplicative inverses.



- In $y = mx + b$ form, the values of m multiply to get -1..

  Example: $y = 6x + 5$ and
  $y = -\frac{1}{6}x - 3$

- In Standard Form, if you add the product of the x-coefficients to the product of the y-coefficients and get zero.

  Example: $4x + 6y = 4$ and
  $3x - 2y = 5$ because $(4 \cdot 3) + \big(6 \cdot (-2)\big) = 0$

- Also, if one line is vertical (i.e., m is undefined) and one line is horizontal (i.e., $m = 0$).

  Example: $x = 6$ and
  $y = 3$

Pre-Algebra
Probability and Odds

Probability

Probability is a measure of the likelihood that an event will occur. It depends on the number of outcomes that represent the event and the total number of possible outcomes. In equation terms,

$$\boldsymbol{P(event) = \frac{number\ of\ outcomes\ representing\ the\ event}{number\ of\ total\ possible\ outcomes}}$$

Example 1: The probability of a flipped coin landing as a head is 1/2. There are two equally likely events when a coin is flipped – it will show a head or it will show a tail. So, there is one chance out of two that the coin will show a head when it lands.

$$P(head) = \frac{1\ outcome\ of\ a\ head}{2\ total\ possible\ outcomes} = \frac{1}{2}$$

Example 2: In a jar, there are 15 blue marbles, 10 red marbles and 7 green marbles. What is the probability of selecting a red marble from the jar? In this example, there are 32 total marbles, 10 of which are red, so there is a 10/32 (or, when reduced, 5/16) probability of selecting a red marble.

$$P(red\ marble) = \frac{10\ red\ marbles}{32\ total\ marbles} = \frac{10}{32} = \frac{5}{16}$$

Odds

Odds are similar to probability, except that we measure the number of chances that an event will occur relative to the number of chances that the event will not occur.

$$\boldsymbol{Odds(event) = \frac{number\ of\ outcomes\ representing\ the\ event}{number\ of\ outcomes\ NOT\ representing\ the\ event}}$$

In the above examples,

$$Odds(head) = \frac{1\ outcome\ of\ a\ head}{1\ outcome\ of\ a\ tail} = \frac{1}{1} \qquad Odds(red\ marble) = \frac{10\ red\ marbles}{22\ other\ marbles} = \frac{10}{22} = \frac{5}{11}$$

- Note that the numerator and the denominator in an odds calculation add to the total number of possible outcomes in the denominator of the corresponding probability calculation.

- To the beginning student, the concept of odds is not as intuitive as the concept of probabilities; however, they are used extensively in some environments.

Pre-Algebra
Probability with Dice

Single Die

Probability with a single die is based on the number of chances of an event out of 6 possible outcomes on the die. For example:

$$P(2) = \frac{1}{6} \qquad P(odd\ number) = \frac{3}{6} = \frac{1}{2} \qquad P(number < 5) = \frac{4}{6} = \frac{2}{3}$$

Two Dice

Probability with two dice is based on the number of chances of an event out of 36 possible outcomes on the dice. The following table of results when rolling 2 dice is helpful in this regard:

	1st Die					
2nd Die	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The probability of rolling a number with two dice is the number of times that number occurs in the table, divided by 36. Here are the probabilities for all numbers 2 to 12.

$P(2) = \frac{1}{36}$ $\quad P(5) = \frac{4}{36} = \frac{1}{9}$ $\quad P(8) = \frac{5}{36}$ $\quad P(11) = \frac{2}{36} = \frac{1}{18}$

$P(3) = \frac{2}{36} = \frac{1}{18}$ $\quad P(6) = \frac{5}{36}$ $\quad P(9) = \frac{4}{36} = \frac{1}{9}$ $\quad P(12) = \frac{1}{36}$

$P(4) = \frac{3}{36} = \frac{1}{12}$ $\quad P(7) = \frac{6}{36} = \frac{1}{6}$ $\quad P(10) = \frac{3}{36} = \frac{1}{12}$

$$P(odd\ number) = \frac{18}{36} = \frac{1}{2} \qquad P(number\ divisible\ by\ 3) = \frac{2+5+4+1}{36} = \frac{12}{36} = \frac{1}{3}$$

$$P(even\ number) = \frac{18}{36} = \frac{1}{2} \qquad P(number\ divisible\ by\ 4) = \frac{3+5+1}{36} = \frac{9}{36} = \frac{1}{4}$$

$$P(number\ divisible\ by\ 6) = \frac{5+1}{36} = \frac{6}{36} = \frac{1}{6}$$

Pre-Algebra
Statistical Measures

Statistical measures help describe a set of data. A definition of a number of these is provided in the table below:

Concept	Description	Calculation	Example 1	Example 2
Data Set	Numbers		35, 35, 37, 38, 45	15, 20, 20, 22, 25, 54
Mean	Average	Add the values and divide the total by the number of values	$\frac{35+35+37+38+45}{5}=38$	$\frac{15+18+22+22+25+54}{6}=26$
Median(1)	Middle	Arrange the values from low to high and take the middle value(1)	37	21(1)
Mode	Most	The value that appears most often in the data set	35	20
Range	Size	The difference between the highest and lowest values in the data set	45 – 35 = 10	54 – 15 = 39
Outliers(2)	Oddballs	Values that look very different from the other values in the data set	none	54

Notes:

(1) If there are an even number of values, the median is the average of the two middle values. In Example 2, the median is 21, which is the average of 20 and 22.

(2) The question of what constitutes an outlier is not always clear. Although statisticians seek to minimize subjectivity in the definition of outliers, different analysts may choose different criteria for the same data set.

Pre-Algebra
Stem-and-Leaf Plots

A **Stem-and-Leaf Plot** is a way to organize data. It is useful for small sets of data only. For large sets of data, the technique becomes cumbersome. From a stem-and-leaf plot, it is relatively easy to calculate the key statistical values:

- The minimum value.
- The first quartile.
- The median.
- The third quartile.
- The maximum value.

In addition, from the maximum value and the minimum value, it is possible to calculate the range (which is the difference between the two values).

In a stem-and-leaf plot,

- The data are arranged in a two-column chart.
- Each data value is split into two separate parts, a stem and a leaf.
- The stems are placed to the left of the vertical line. In a common version of the plot, the stems are all of the digits of each value except the "ones digit." A separate row is created for each stem, in numerical order.
- The leaves are placed to the right of the vertical line. In the version of the plot mentioned above, the leaves are the "ones digits" of the values corresponding to each stem. The values in each row are placed in numerical order, separated by commas.
- Often, a key is placed on the plot which illustrates how the individual data are split between the leaf and the stem.

Example: Consider the following data set:

$$\{37, 41, 28, 52, 56, 32, 37, 42, 26, 37, 40, 37, 51, 48\}$$

The stem and leaf plot would be:

Stem	Leaves
2	6, 8
3	2, 7, 7, 7, 7
4	0, 1, 2, 8
5	1, 2, 6

Notice that in stem-and-leaf form, the data are ordered. They are also organized in a way that makes calculating statistical measures like the median, mode, first and third quartiles, and range are relatively easy.

Pre-Algebra
Box-and-Whisker Plots

A **Box-and-Whisker Plot** is a way to present information about a data set. The following information about a set of data is provided by a box-and-whisker plot:

- The minimum value.
- The first quartile.
- The median.
- The third quartile.
- The maximum value.

> In addition, from the maximum value and the minimum value, it is possible to calculate the range (which is the difference between the two values).

A plot with small boxes and big whiskers tells us that the data is spread out, whereas a plot with large boxes and small whiskers tells us that the data is more tightly grouped.

A Few Points about Discreet Data

When data sets consist of a small number of observations, questions arise about the precise definitions of the above terms in some circumstances. In particular,

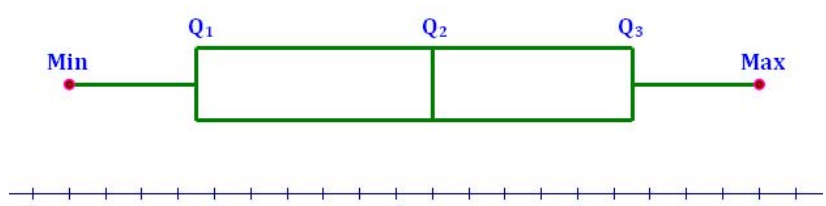
- **Median:** When values are arranged in order, the median is the middle value. If the number of values is even, there are two middle values; the median is their mean.
- **First Quartile:** The first quartile is the median of the lower half of the values. When there are an odd number of values overall, the median is omitted from this calculation. The first quartile is sometimes referred to as the lower quartile.
- **Third Quartile:** The third quartile is the median of the upper half of the values. When there are an odd number of values overall, the median is omitted from this calculation. The third quartile is sometimes referred to as the upper quartile.

Example:
Create a box-and-whisker plot for the following ordered data set:

$$\{6, 8, 11, 13, 16, 20, 20, 23, 25\}$$

There are 9 values

- Minimum value: 6
- Q_1 – First quartile: 9.5
- Q_2 – Median: 16
- Q_3 – Third quartile: 21.5
- Maximum value: 25



Pre-Algebra
Distance Between Points

Distance measures how far apart two things are. The distance between two points can be measured in any number of dimensions, and is defined as the length of the line connecting the two points. Distance is always a positive number.

1-Dimensional Distance

In one dimension the distance between two points is determined simply by subtracting the coordinates of the points.

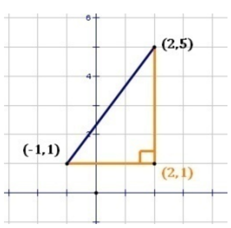
Example: In this segment, the distance between -2 and 5 is calculated as: $5 - (-2) = 7.$

2-Dimensional Distance

In two dimensions, the distance between two points can be calculated by considering the line between them to be the hypotenuse of a right triangle. To determine the length of this line:

- Calculate the difference in the x-coordinates of the points
- Calculate the difference in the y-coordinates of the points
- Use the Pythagorean Theorem.

This process is illustrated below, using the variable ***"d"*** for distance.



Example: Find the distance between (-1,1) and (2,5). Based on the illustration to the left:

x-coordinate difference: $2 - (-1) = 3.$

y-coordinate difference: $5 - 1 = 4.$

Then, the distance is calculated using the formula: $d^2 = (3^2 + 4^2) = (9 + 16) = 25$

So, $\boldsymbol{d = 5}$

If we define two points generally as ***(x_1, y_1)*** and ***(x_2, y_2)***, then a 2-dimensional distance formula would be:

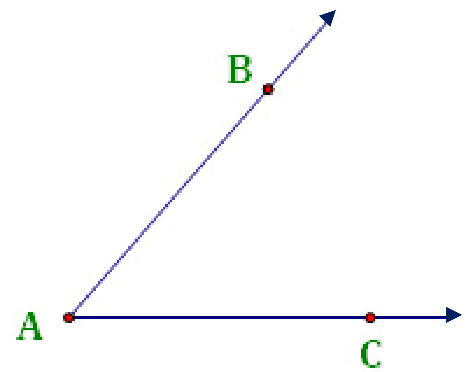
$$\boldsymbol{distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Geometry
Angles

Parts of an Angle

An **angle** consists of two rays with a common endpoint (or, initial point).

- Each ray is a **side** of the angle.
- The common endpoint is called the **vertex** of the angle.



Naming Angles

Angles can be named in one of two ways:

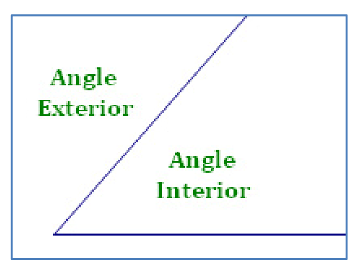
- **Point-vertex-point method.** In this method, the angle is named from a point on one ray, the vertex, and a point on the other ray. This is the most unambiguous method of naming an angle, and is useful in diagrams with multiple angles sharing the same vertex. In the above figure, the angle shown could be named $\angle BAC$ or $\angle CAB$.
- **Vertex method.** In cases where it is not ambiguous, an angle can be named based solely on its vertex. In the above figure, the angle could be named $\angle A$.

Measure of an Angle

There are two conventions for measuring the size of an angle:

- **In degrees**. The symbol for degrees is ^{o}. There are 360^{o} in a full circle. The angle above measures approximately **45^{o}** (one-eighth of a circle).
- **In radians**. There are 2π radians in a complete circle. The angle above measures approximately $\frac{1}{4}\pi$ **radians**.

Some Terms Relating to Angles



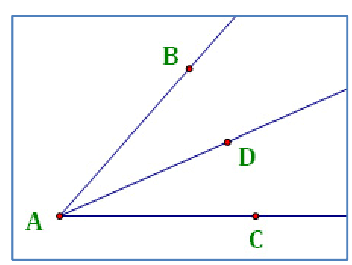
Angle interior is the area between the rays.

Angle exterior is the area not between the rays.

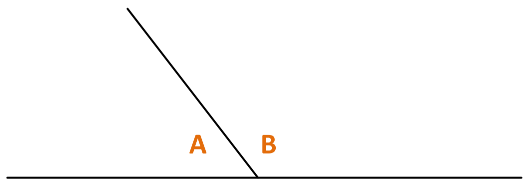
Adjacent angles are angles that share a ray for a side. $\angle BAD$ and $\angle DAC$ in the figure at right are adjacent angles.

Congruent angles area angles with the same measure.

Angle bisector is a ray that divides the angle into two congruent angles. Ray $\overrightarrow{AD}$ bisects $\angle BAC$ in the figure at right.



Pre-Algebra
Types of Angles

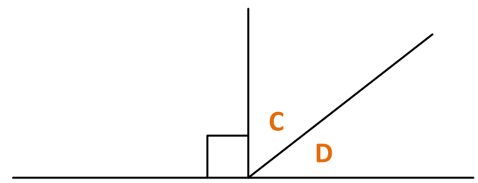


Supplementary Angles

Angles A and B are supplementary.

Angles A and B form a linear pair.

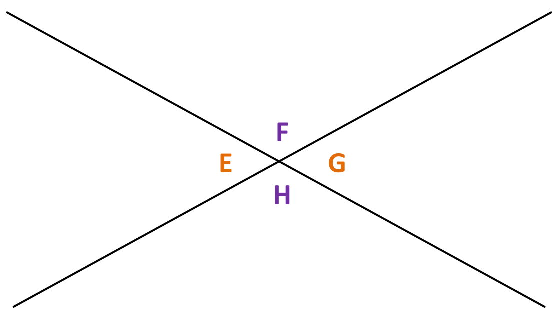
$m\angle A + m\angle B = 180^0$



Complementary Angles

Angles C and D are complementary.

$m\angle C + m\angle D = 90^0$



Vertical Angles

Angles which are opposite each other when two lines cross are vertical angles.

Angles E and G are vertical angles.
Angles F and H are vertical angles.

$m\angle E = m\angle G \quad and \quad m\angle F = m\angle H$

In addition, each angle is supplementary to the two angles adjacent to it. For example:

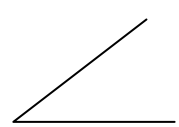
Angle E is supplementary to Angles F and H.

An acute angle is one that is less than 90^{o}. In the illustration above, angles E and G are acute angles.

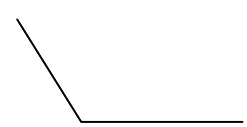
A right angle is one that is exactly 90^{o}.

An obtuse angle is one that is greater than 90^{o}. In the illustration above, angles F and H are obtuse angles.

A straight angle is one that is exactly 180^{o}.



Acute



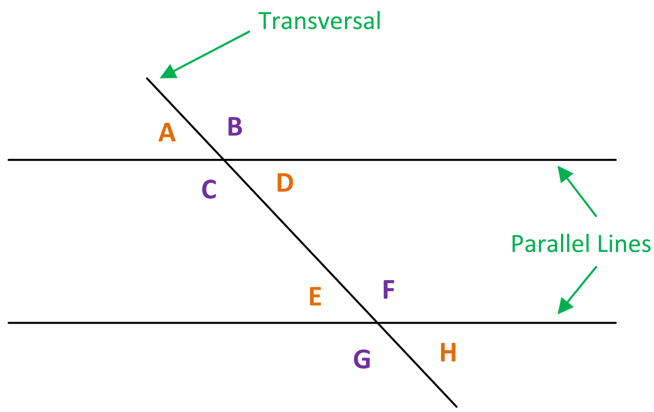
Obtuse

Right

Straight

Pre-Algebra

Parallel Lines and Transversals



Alternate: refers to angles that are on opposite sides of the transversal.

Consecutive: refers to angles that are on the same side of the transversal.

Interior: refers to angles that are between the parallel lines.

Exterior: refers to angles that are outside the parallel lines.

Corresponding Angles

Corresponding Angles are angles in the same location relative to the parallel lines and the transversal. For example, the angles on top of the parallel lines and left of the transversal (i.e., top left) are corresponding angles.

Angles **A** and **E (top left)** are **Corresponding Angles**. So are angle pairs **B** and **F (top right)**, **C** and **G (bottom left)**, and **D** and **H (bottom right)**. Corresponding angles are congruent.

Alternate Interior Angles

Angles **D** and **E** are **Alternate Interior Angles**. Angles **C** and **F** are also alternate interior angles. Alternate interior angles are congruent.

Alternate Exterior Angles

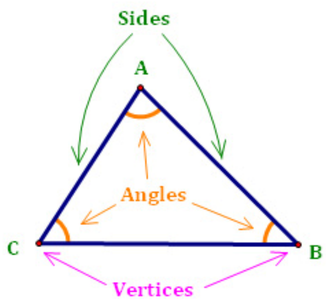
Angles **A** and **H** are **Alternate Exterior Angles**. Angles **B** and **G** are also alternate exterior angles. Alternate exterior angles are congruent.

Consecutive Interior Angles

Angles **C** and **E** are **Consecutive Interior Angles**. Angles **D** and **F** are also consecutive interior angles. Alternate exterior angles are congruent.

Note that angles ***A****,* ***D****,* ***E****, and* ***H*** *are congruent, and angles* ***B****,* ***C****,* ***F****, and* ***G*** *are congruent. In addition, each of the angles in the first group are supplementary to each of the angles in the second group.*

Pre-Algebra
What Makes a Triangle?

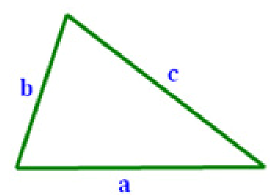


A triangle is a closed path with 3 vertices (points), 3 sides and 3 angles. In the triangle at left,

- The vertices are points A, B, and C.
- the sides are $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.
- The angles are the located inside the triangle at the vertices. They are $\angle A, \angle B$, and $\angle C$.

Sizes of Sides

Triangle Inequality: The sum of the lengths of any two sides of a triangle is greater than the length of the third side. This is a crucial element in deciding whether segments of any 3 lengths can form a triangle.



$$\boldsymbol{a} + \boldsymbol{b} > c \quad \textbf{\textit{and}} \quad \boldsymbol{b} + \boldsymbol{c} > a \quad and \quad \boldsymbol{c} + \boldsymbol{a} > b$$

Tip: Another way to look at this is to arrange the sides in order of size, smallest to largest. Then, if the largest side is less than the sum of the other two sides, the three sides make a valid triangle.

Sum of Angles

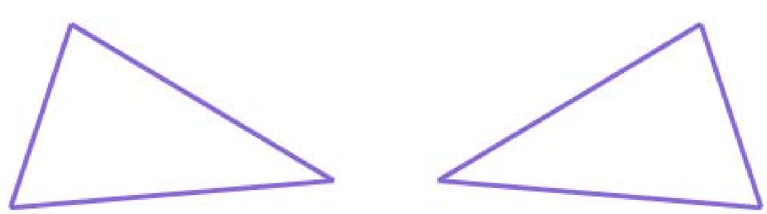
In a triangle, the sum of the measures of the 3 angles is 180^0. In the triangle above left on this page, that means that:

$$m\angle A + \angle B + \angle C = 180^0$$

This is a very important fact in Geometry; you will be required to recall this on almost a daily basis in working with triangles.

Congruent Triangles

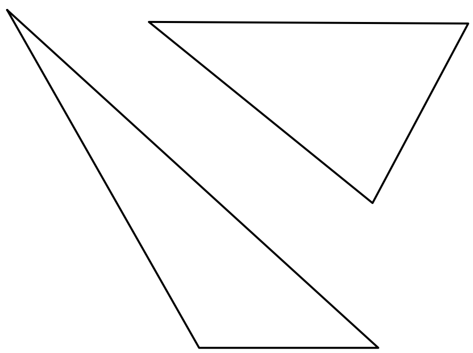
Two triangles are **congruent** if they have the same exact dimensions. That is, all of the sides are the same length and all of the angles have the same measure. They do not need to be facing the same way, they just need to have the same measures of sides and angles.



Pre-Algebra
Types of Triangles

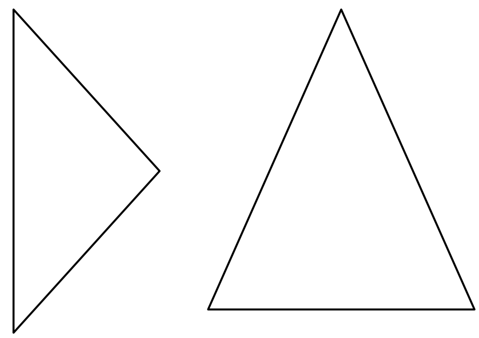
Scalene

A Scalene Triangle has 3 sides of different lengths. Because the sides are of different lengths, the angles must also be of different measures.



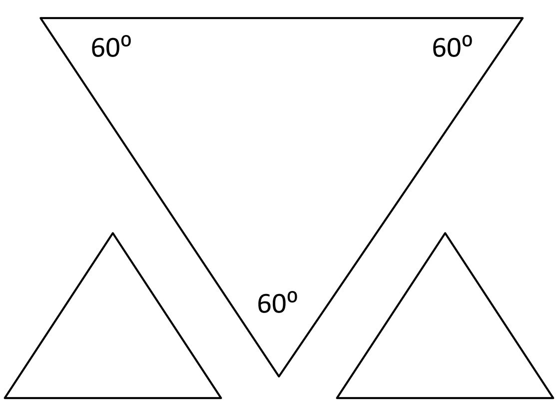
Isosceles

An Isosceles Triangle has 2 sides the same length (i.e., congruent). Because two sides are congruent, two angles must also be congruent.



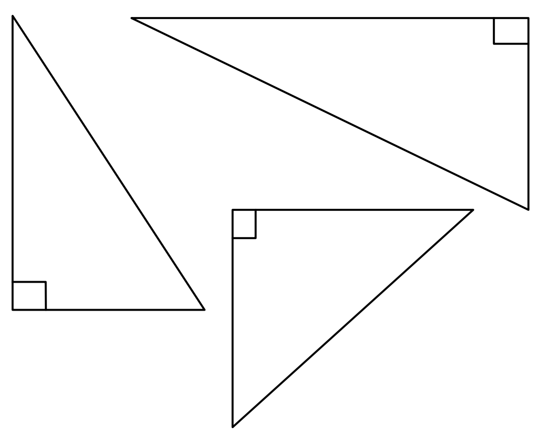
Equilateral

An Equilateral Triangle has all 3 sides the same length (i.e., congruent). Because all 3 sides are congruent, all 3 angles must also be congruent. This requires each angle to be 60^o.



Right

A Right Triangle is one that contains a 90^o angle. It may be scalene or isosceles, but cannot be equilateral. Right triangles have sides that meet the requirements of the Pythagorean Theorem.



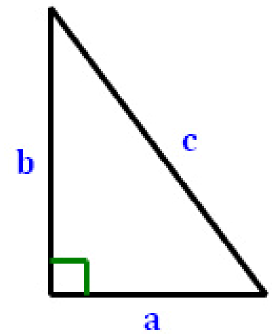
Pre-Algebra
Pythagorean Theorem

In a right triangle, the Pythagorean Theorem says:

$$\boldsymbol{a^2 + b^2 = c^2}$$

where,

- $\boldsymbol{a}$ and $\boldsymbol{b}$ are the lengths of the legs of a right triangle, and
- $\boldsymbol{c}$ is the length of the hypotenuse.

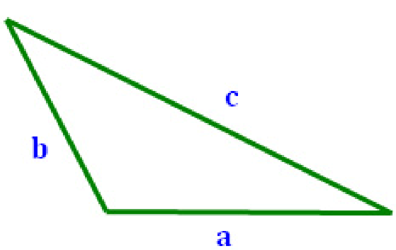


Right, Acute, or Obtuse Triangle?

In addition to allowing the solution of right triangles, the Pythagorean Formula can be used to determine whether a triangle is a right triangle, an acute triangle, or an obtuse triangle.

To determine whether a triangle is obtuse, right, or acute:

- Arrange the lengths of the sides from low to high; call them a, b, and c, in increasing order
- Calculate: a^2, b^2, and c^2.
- Compare: $a^2 + b^2 \text{ vs. } c^2$
- Use the illustrations below to determine which type of triangle you have.



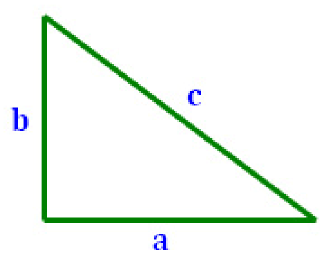
Obtuse Triangle

$$\boldsymbol{a^2 + b^2 < c^2}$$

Example:

Triangle with sides: 7, 9, 12

$$7^2 + 9^2 \;\; vs. \;\; 12^2$$
$$49 + 81 < 144$$
$$\rightarrow \; \boldsymbol{Obtuse\ Triangle}$$



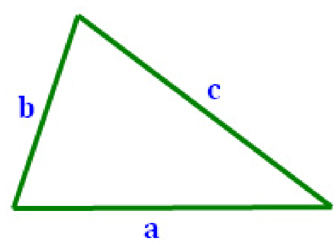
Right Triangle

$$\boldsymbol{a^2 + b^2 = c^2}$$

Example:

Triangle with sides: 6, 8, 10

$$6^2 + 8^2 \;\; vs. \;\; 10^2$$
$$36 + 64 = 100$$
$$\rightarrow \; \boldsymbol{Right\ Triangle}$$



Acute Triangle

$$\boldsymbol{a^2 + b^2 > c^2}$$

Example:

Triangle with sides: 5, 8, 9

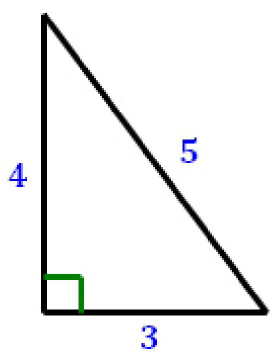
$$5^2 + 8^2 \;\; vs. \;\; 9^2$$
$$25 + 64 > 81$$
$$\rightarrow \; \boldsymbol{Acute\ Triangle}$$

Pre-Algebra
Pythagorean Triples

Pythagorean Theorem: $\boldsymbol{a^2 + b^2 = c^2}$

Pythagorean triples are sets of 3 positive integers that meet the requirements of the Pythagorean Theorem. Because these sets of integers provide "pretty" solutions to geometry problems, they are a favorite of geometry books and teachers. Knowing what triples exist can help the student quickly identify solutions to problems that might otherwise take considerable time to solve.

3-4-5 Triangle Family



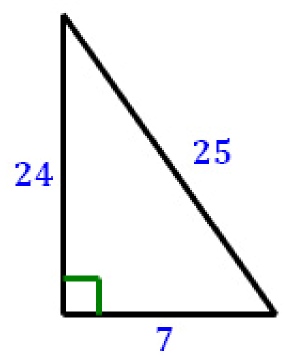
Sample Triples

3-4-5
6-8-10
9-12-15
12-16-20
30-40-50

$$\mathbf{3^2 + 4^2 = 5^2}$$

$$9 + 16 = 25$$

7-24-25 Triangle Family



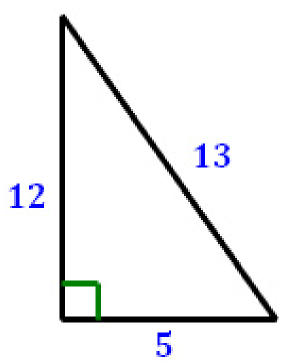
Sample Triples

7-24-25
14-48-50
21-72-75
. . .
70-240-250

$$\mathbf{7^2 + 24^2 = 25^2}$$

$$49 + 576 = 625$$

5-12-13 Triangle Family



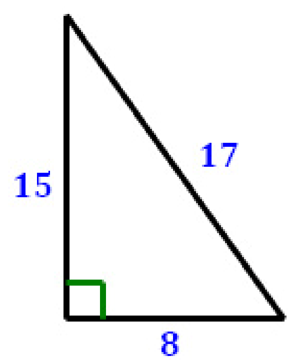
Sample Triples

5-12-13
10-24-26
15-36-39
. . .
50-120-130

$$\mathbf{5^2 + 12^2 = 13^2}$$

$$25 + 144 = 169$$

8-15-17 Triangle Family



Sample Triples

8-15-17
16-30-34
24-45-51
. . .
80-150-170

$$\mathbf{8 + 15^2 = 17^2}$$

$$64 + 225 = 289$$

Pre-Algebra
Ratios and Proportions

Ratios are fractions that relate two items. For example, in baseball, the ratio of the number of a batter's hit to the number of his at-bats provides his batting average. That is:

$$\frac{hits}{at\ bats} = batting\ average$$

Proportions are relationships between ratios. For example, if two batters have the same batting average, then:

$$\frac{Player\ A\ \text{"}hits\text{"}}{Player\ A\ \text{"}at\ bats\text{"}} = \frac{Player\ B\ \text{"}hits\text{"}}{Player\ B\ \text{"}at\ bats\text{"}}$$

So, for example, if we know that $Player\ A$ has 26 hits in 80 at-bats, and $Player\ B$, with the same batting average has 120 at-bats, let's calculate $x = the\ number\ of\ Player\ B's\ hits$. From the proportion above, we know that:

$$\frac{26}{80} = \frac{x}{120}$$

Proportions are usually solved by cross multiplying. In the example,

$$(26)(120) = (80)(x)$$

$$\boldsymbol{x = 39}$$

Ratios Involving Units

When simplifying ratios containing the same units:

- Simplify the fraction.
- Notice that the units disappear. They behave just like factors; if the units exist in the numerator and denominator, the cancel and are not in the answer.

Example:

$$\frac{3\ inches}{12\ inches} = \frac{1}{4}$$

Note: the unit "inches cancel out, so the answer is $\frac{1}{4}$, not $\frac{1}{4}\ inch$.

When simplifying ratios containing different units:

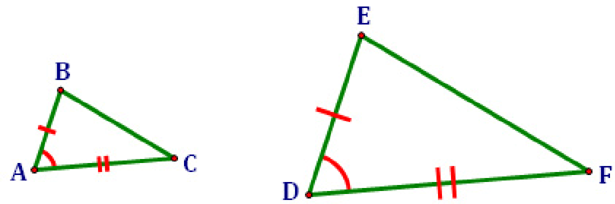
- Adjust the ratio so that the numerator and denominator have the same units.
- Simplify the fraction.
- Notice that the units disappear.

Example:

$$\frac{3\ inches}{2\ feet} = \frac{3\ inches}{(2\ feet) \cdot (12\ inches/foot)} = \frac{3\ inches}{24\ inches} = \frac{1}{8}$$

Pre-Algebra
Similar Triangles

Similar Triangle Parts



In similar triangles,

- Corresponding sides are proportional.
- Corresponding angles are congruent.

In working with similar triangles is crucial to line up corresponding vertices. Once this is done, the rest of the picture becomes clear. In the picture above,

- Point A corresponds to Point D.
- Point B corresponds to Point E.
- Point C corresponds to Point F.

Naming Similar Triangles

Based on the above Correspondences, we can say:

$$\Delta ABC \sim \Delta DEF \quad \text{or} \quad \Delta BCA \sim \Delta EFD \quad \text{or} \quad \Delta CAB \sim \Delta FDE \quad \text{or}$$
$$\Delta ACB \sim \Delta DFE \quad \text{or} \quad \Delta BAC \sim \Delta EDF \quad \text{or} \quad \Delta CBA \sim \Delta FED$$

All of these are correct because they match corresponding parts in the naming. Each of these similarities implies the following relationships between parts of the two triangles:

Angles: $\angle A \cong \angle D \quad \text{and} \quad \angle B \cong \angle E \quad \text{and} \quad \angle C \cong \angle F$

Sides: $$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

The relationships among the sides of the triangles allow us to calculate information about some of the sides based on information about other sides. For example, if we know that $AB = 6$, $DE = 12$ and $CA = 10$, we can calculate $x = FD$ as follows:

$$\frac{AB}{DE} = \frac{CA}{FD} \quad \Longrightarrow \quad \frac{6}{12} = \frac{10}{x}$$

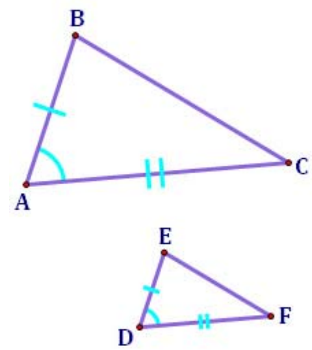
$$6x = 120$$

$$\boldsymbol{x = 20}$$

Pre-Algebra
Proportion Tables for Similar Triangles

Setting Up a Table of Proportions

It is often useful to set up a table to identify the proper proportions in a similarity. Consider the figure to the right. The table might look something like this:



Triangle	Left Side	Right Side	Bottom Side
Top Δ	AB	BC	CA
Bottom Δ	DE	EF	FD

The purpose of a table like this is to organize the information you have about the similar triangles so that you can readily develop the proportions you need.

Developing the Proportions

To develop proportions from the table:

- Extract the columns needed from the table:

AB	BC
DE	EF

- Eliminate the table lines.
- Replace the horizontal lines with "division lines."
- Put an equal sign between the two resulting fractions:

$$\frac{AB}{DE} = \frac{BC}{EF}$$

Also from the above table,

$$\frac{AB}{DE} = \frac{CA}{FD}$$

$$\frac{BC}{EF} = \frac{CA}{FD}$$

Solving for the unknown length of a side:

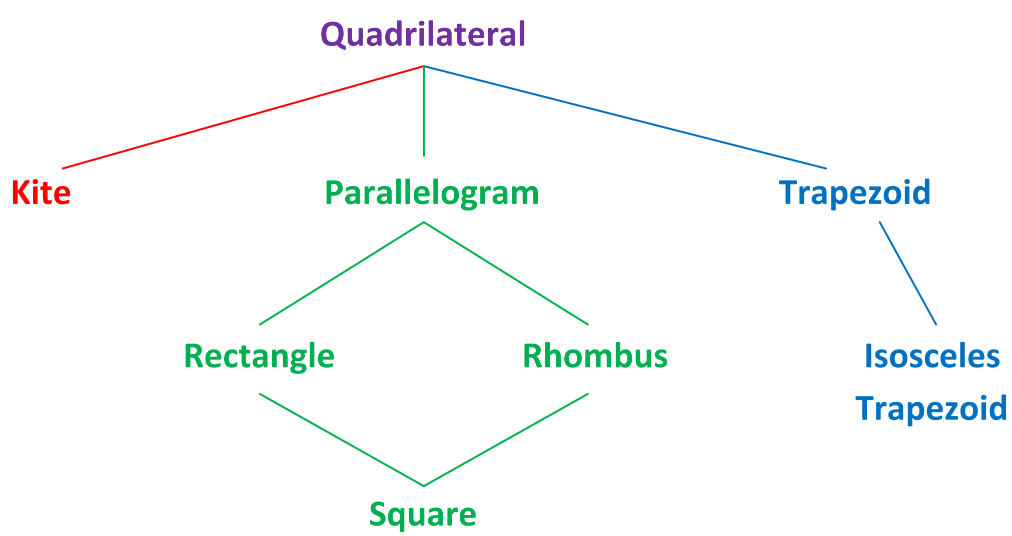
You can extract any two columns you like from the table. Usually, you will have information on lengths of three of the sides and will be asked to calculate a fourth.

Look in the table for the columns that contain the 4 sides in question, and then set up your proportion. Substitute known values into the proportion, and solve for the remaining variable.

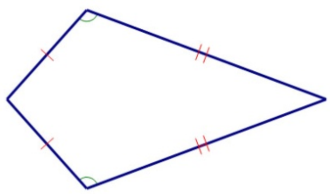
Pre-Algebra
Definitions of Quadrilaterals

Name	Definition
Quadrilateral	A polygon with 4 sides.
Kite	A quadrilateral with two consecutive pairs of congruent sides, but with opposite sides not congruent.
Trapezoid	A quadrilateral with exactly one pair of parallel sides.
Isosceles Trapezoid	A trapezoid with congruent legs.
Parallelogram	A quadrilateral with both pairs of opposite sides parallel.
Rectangle	A parallelogram with all angles congruent (i.e., right angles).
Rhombus	A parallelogram with all sides congruent.
Square	A quadrilateral with all sides congruent and all angles congruent.

Quadrilateral Tree:

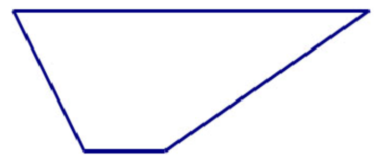


Pre-Algebra
Figures of Quadrilaterals



Kite

- 2 consecutive pairs of congruent sides
- 1 pair of congruent opposite angles
- Diagonals perpendicular

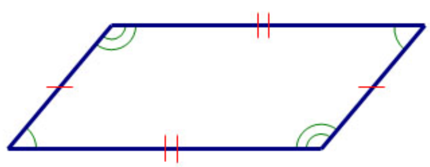


Trapezoid

- 1 pair of parallel sides (called “bases”)
- Angles on the same “side” of the bases are supplementary

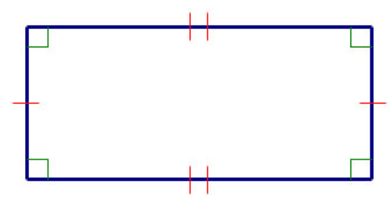
Isosceles Trapezoid

- 1 pair of parallel sides
- Congruent legs
- 2 pair of congruent base angles
- Diagonals congruent



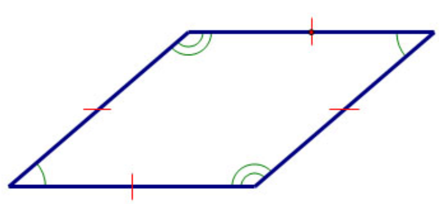
Parallelogram

- Both pairs of opposite sides parallel
- Both pairs of opposite sides congruent
- Both pairs of opposite angles congruent
- Consecutive angles supplementary
- Diagonals bisect each other



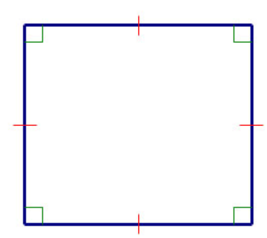
Rectangle

- Parallelogram with all angles congruent (i.e., right angles)
- Diagonals congruent



Rhombus

- Parallelogram with all sides congruent
- Diagonals perpendicular
- Each diagonal bisects a pair of opposite angles



Square

- Both a Rhombus and a Rectangle
- All angles congruent (i.e., right angles)
- All sides congruent

Pre-Algebra
Characteristics of Parallelograms

Characteristic	Square	Rhombus	Rectangle	Parallelogram
2 pair of parallel sides	✓	✓	✓	✓
Opposite sides are congruent	✓	✓	✓	✓
Opposite angles are congruent	✓	✓	✓	✓
Consecutive angles are supplementary	✓	✓	✓	✓
Diagonals bisect each other	✓	✓	✓	✓
All 4 angles are congruent (i.e., right angles)	✓		✓	
Diagonals are congruent	✓		✓	
All 4 sides are congruent	✓	✓		
Diagonals are perpendicular	✓	✓		
Each diagonal bisects a pair of opposite angles	✓	✓		

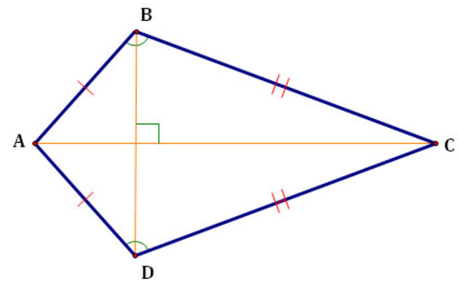
Notes: Red ✓-marks are conditions sufficient to prove the quadrilateral is of the type specified.
Green ✓-marks are conditions sufficient to prove the quadrilateral is of the type specified if the quadrilateral is a parallelogram.

Pre-Algebra
Kites and Trapezoids

Facts about a Kite

To prove a quadrilateral is a kite, prove:

- It has two pair of congruent sides.
- Opposite sides are not congruent.



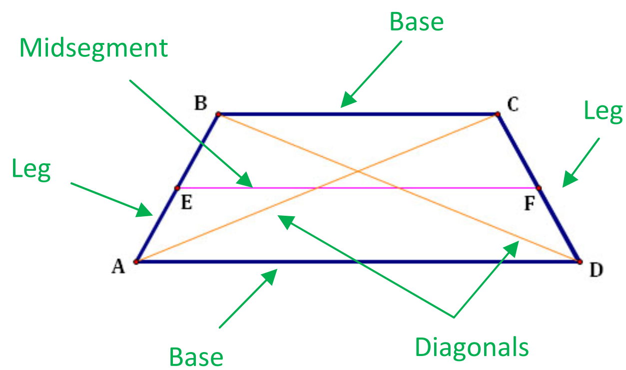
Also, if a quadrilateral is a kite, then:

- Its diagonals are perpendicular
- It has exactly one pair of congruent opposite angles.

Parts of a Trapezoid

Trapezoid ABCD has the following parts:

- $\overline{AD}$ and $\overline{BC}$ are **bases**.
- $\overline{AB}$ and $\overline{CD}$ are **legs**.
- $\overline{EF}$ is the **midsegment**.
- $\overline{AC}$ and $\overline{BD}$ are **diagonals**.
- Angles A and D form a pair of **base angles**.
- Angles B and C form a pair of **base angles**.



Trapezoid Midsegment Theorem

The **midsegment** of a trapezoid is parallel to each of its bases and: $EF = \frac{1}{2}\,(AD + BC)$.

Proving a Quadrilateral is an Isosceles Trapezoid

To prove a quadrilateral is an isosceles trapezoid, prove any of the following conditions:

1. It is a trapezoid and has a pair of congruent legs. (definition of isosceles trapezoid)
2. It is a trapezoid and has a pair of congruent base angles.
3. It is a trapezoid and its diagonals are congruent.

Pre-Algebra
Introduction to Transformation

A **Transformation** is a mapping of the pre-image of a geometric figure onto an image that retains key characteristics of the pre-image.

Definitions

The **Pre-Image** is the geometric figure before it has been transformed.

The **Image** is the geometric figure after it has been transformed.

A **mapping** is an association between objects. Transformations are types of mappings. In the figures below, we say $ABCD$ *is mapped onto* $A'B'C'D'$, or $ABCD \longrightarrow A'B'C'D'$. The order of the vertices is critical to a properly named mapping.

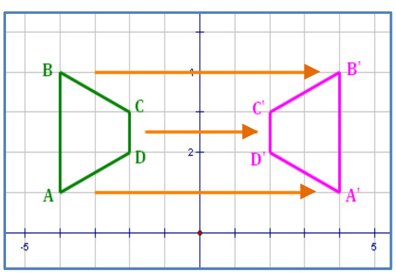
An **Isometry** is a one-to-one mapping that preserves lengths. Transformations that are isometries (i.e., preserve length) are called **rigid transformations**.

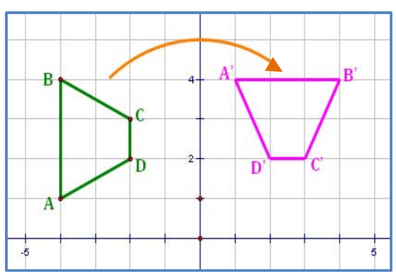
Isometric Transformations

Reflection is flipping a figure across a line called a "mirror." The figure retains its size and shape, but appears "backwards" after the reflection.

Rotation is turning a figure around a point. Rotated figures retain their size and shape, but not their orientation.

Translation is sliding a figure in the plane so that it changes location but retains its shape, size and orientation.





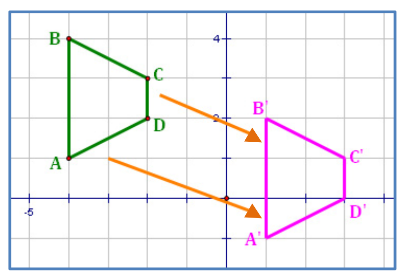


Table of Characteristics of Isometric Transformations

Transformation	Reflection	Rotation	Translation
Isometry (Retains Lengths)?	Yes	Yes	Yes
Retains Angles?	Yes	Yes	Yes
Retains Orientation to Axes?	No	No	Yes

Pre-Algebra
Introduction to Transformation (cont'd)

Transformation of a Point

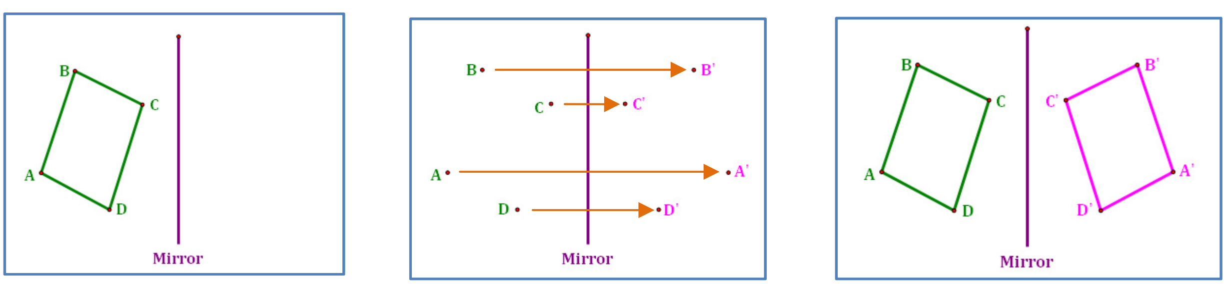
A point is the easiest object to transform. Simply reflect, rotate or translate it following the rules for the transformation selected. By transforming key points first, any transformation becomes much easier.

Transformation of a Geometric Figure

To transform any geometric figure, it is only necessary to transform the items that define the figure, and then re-form it. For example:

- To transform a **line segment**, transform its two endpoints, and then connect the resulting images with a line segment.
- To transform a **ray**, transform the initial point and any other point on the ray, and then construct a ray using the resulting images.
- To transform a **line**, transform any two points on the line, and then fit a line through the resulting images.
- To transform a **polygon**, transform each of its vertices, and then connect the resulting images with line segments.
- To transform a **circle**, transform its center and, if necessary, its radius. From the resulting images, construct the image circle.
- To transform **other conic sections (parabolas, ellipses and hyperbolas)**, transform the foci, vertices and/or directrix. From the resulting images, construct the image conic section.

Example: Reflect Quadrilateral ABCD

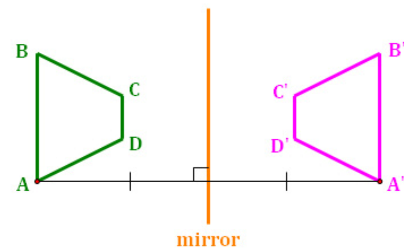


Pre-Algebra
Reflection

Definitions

Reflection is flipping a figure across a mirror.

The **Line of Reflection** is the mirror through which the reflection takes place.



Note that:

- The line segment connecting corresponding points in the image and pre-image is bisected by the mirror.
- The line segment connecting corresponding points in the image and pre-image is perpendicular to the mirror.

Reflection through an Axis or the Line $y = x$

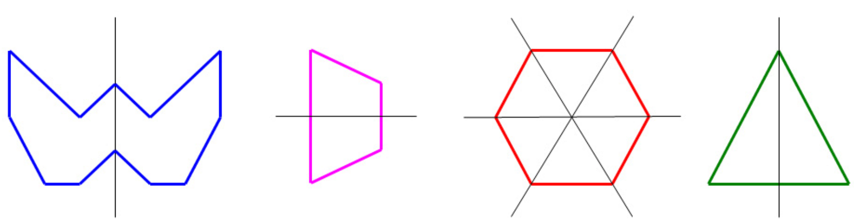
Reflection of the point (a, b) through the x- or y-*axis* or the line $y = x$ gives the following results:

Pre-Image Point	Mirror Line	Image Point
(a, b)	x-axis	(a, -b)
(a, b)	y-axis	(-a, b)
(a, b)	the line: $y = x$	(-a, -b)

If you forget the above table, start with the point $(3, 2)$ on a set of coordinate axes. Reflect the point through the selected line and see which set of "a, b" coordinates works.

Line of Symmetry

A **Line of Symmetry** is any line through which a figure can be mapped onto itself. The thin black lines in the following figures show their axes of symmetry:



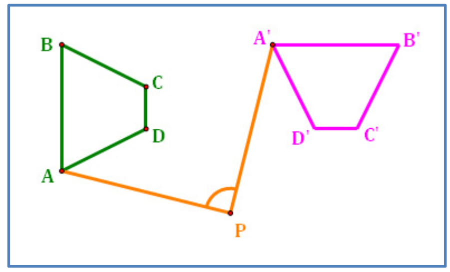
Pre-Algebra
Rotation

Definitions

Rotation is turning a figure by an angle about a fixed point.

The **Center of Rotation** is the point about which the figure is rotated. Point ***P***, at right, is the center of rotation.

The **Angle of Rotation** determines the extent of the rotation. The angle is formed by the rays that connect the center of rotation to the pre-image and the image of the rotation. Angle ***P***, at right, is the angle of rotation. Though shown only for Point A, the angle is the same for any of the figure's 4 vertices.



Note: In performing rotations, it is important to indicate the direction of the rotation – clockwise or counterclockwise.

Rotation about the Origin

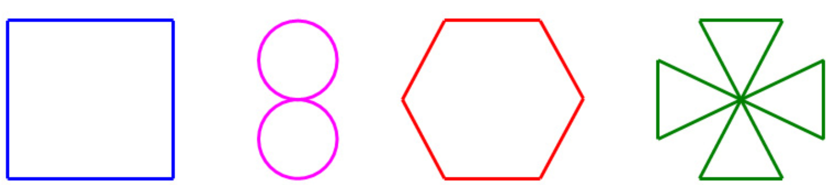
Rotation of the point (a, b) about the origin (0, 0) gives the following results:

Pre-Image Point	Clockwise Rotation	Counterclockwise Rotation	Image Point
(a, b)	90^0	270^0	(b, -a)
(a, b)	180^0	180^0	(-a, -b)
(a, b)	270^0	90^0	(-b, a)
(a, b)	360^0	360^0	(a, b)

If you forget the above table, start with the point $(3, 2)$ on a set of coordinate axes. Rotate the point by the selected angle and see which set of "a, b" coordinates works.

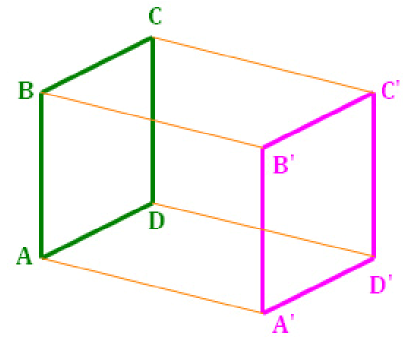
Rotational Symmetry

A figure in a plane has **Rotational Symmetry** if it can be mapped onto itself by a rotation of 180^0 or less. Any regular polygon has rotational symmetry, as does a circle. Here are some examples of figures with rotational symmetry:



Pre-Algebra
Translation

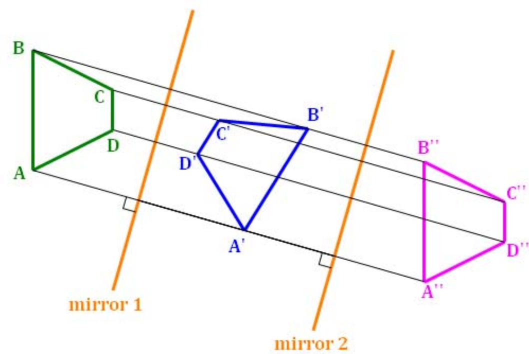
Definitions



Translation is sliding a figure in the plane. Each point in the figure is moved the same distance in the same direction. The result is an image that looks the same as the pre-image in every way, except it has been moved to a different location in the plane.

Each of the four orange line segments in the figure at right has the same length and direction.

When Two Reflections = One Translation



If two mirrors are parallel, then reflection through one of them, followed by a reflection through the second is a translation.

In the figure at right, the black lines show the paths of the two reflections; this is also the path of the resulting translation. Note the following:

- The distance of the resulting translation (e.g., from $\boldsymbol{A}$ to $\boldsymbol{A''}$) is double the distance between the mirrors.
- The black lines of movement are perpendicular to both mirrors.

Defining Translations in the Coordinate Plane (Using Vectors)

A translation moves each point by the same distance in the same direction. In the coordinate plane, this is equivalent to moving each point the same amount in the *x-direction* and the same amount in the *y-direction*. This combination of *x*- and *y-direction* movement is described by a mathematical concept called a **vector**.

In the above figure, translation from $\boldsymbol{A}$ to $\boldsymbol{A''}$ moves *10* in the *x-direction* and the *-3* in the *y-direction*. In vector notation, this is: $\overrightarrow{\boldsymbol{AA''}} = \langle \mathbf{10}, \mathbf{-3} \rangle$. Notice the "half-ray" symbol over the two points and the funny-looking brackets around the movement values.

So, the translation resulting from the two reflections in the above figure moves each point of the pre-image by the vector $\overrightarrow{\boldsymbol{AA''}}$. Every translation can be defined by the vector representing its movement in the coordinate plane.

Pre-Algebra
Compositions

When multiple transformations are combined, the result is called a **Composition of the Transformations**. Two examples of this are:

- Combining two reflections through parallel mirrors to generate a translation (see the previous page).
- Combining a translation and a reflection to generate what is called a **glide reflection**. The glide part of the name refers to translation, which is a kind of gliding of a figure on the plane.

> **Note:** In a **glide reflection**, if the line of reflection is parallel to the direction of the translation, it does not matter whether the reflection or the translation is performed first.

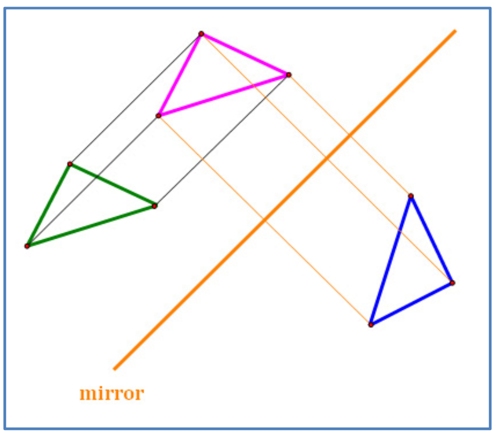


Figure 1: Translation followed by Reflection.

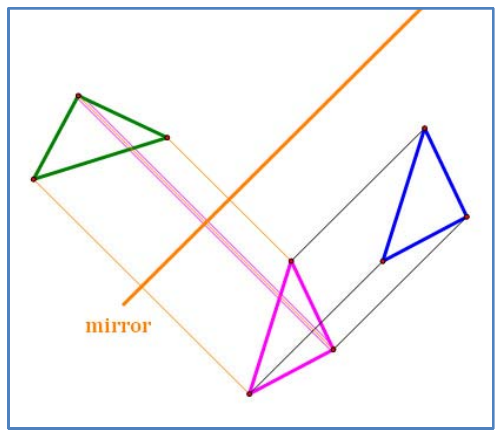


Figure 2: Reflection followed by Translation.

Composition Theorem

The composition of multiple isometries is as Isometry. Put more simply, if transformations that preserve length are combined, the composition will preserve length. This is also true of compositions of transformations that preserve angle measure.

Order of Composition

Order matters in most compositions that involve more than one class of transformation. If you apply multiple transformations of the same kind (e.g., reflection, rotation, or translation), order generally does not matter; however, applying transformations in more than one class may produce different final images if the order is switched.

Pre-Algebra
Polygons - Basics

Basic Definitions

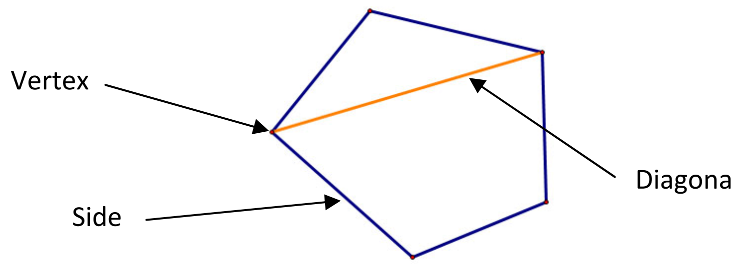
Polygon: a closed path of three or more line segments, where:

- no two sides with a common endpoint are collinear, and
- each segment is connected at its endpoints to exactly two other segments.

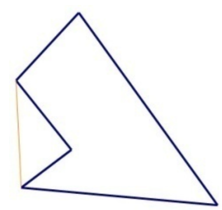
Side: a segment that is connected to other segments (which are also sides) to form a polygon.

Vertex: a point at the intersection of two sides of the polygon. (plural form: **vertices**)

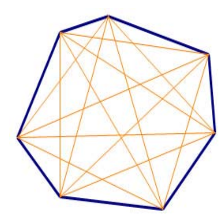
Diagonal: a segment, from one vertex to another, which is not a side.



Concave: A polygon in which it is possible to draw a diagonal "outside" the polygon. (Notice the orange diagonal drawn outside the polygon at right.) Concave polygons actually look like they have a "cave" in them.



Convex: A polygon in which it is <u>not</u> possible to draw a diagonal "outside" the polygon. (Notice that all of the orange diagonals are inside the polygon at right.) Convex polygons appear more "rounded" and do not contain "caves."



Names of Some Common Polygons

Number of Sides	Name of Polygon	Number of Sides	Name of Polygon
3	Triangle	9	Nonagon
4	Quadrilateral	10	Decagon
5	Pentagon	11	Undecagon
6	Hexagon	12	Dodecagon
7	Heptagon	20	Icosagon
8	Octagon	n	n-gon

Names of polygons are generally formed from the Greek language; however, some hybrid forms of Latin and Greek (e.g., undecagon) have crept into common usage.

Pre-Algebra
Polygons – More Definitions

Definitions

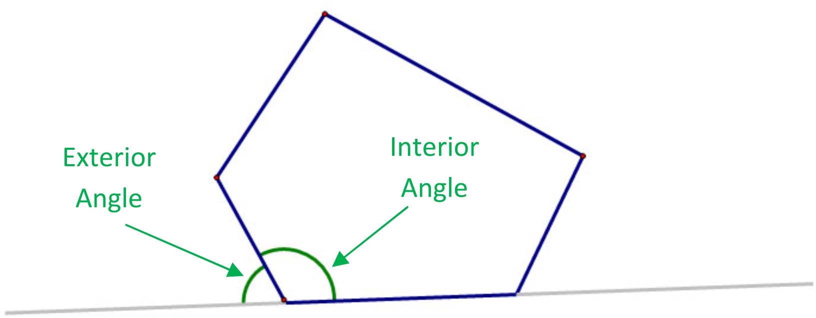
Equilateral: a polygon in which all of the sides are equal in length.

Equiangular: a polygon in which all of the angles have the same measure.

Regular: a polygon which is both equilateral and equiangular. That is, a ***regular polygon*** is one in which all of the sides have the same length and all of the angles have the same measure.

Interior Angle: An angle formed by two sides of a polygon. The angle is inside the polygon.

Exterior Angle: An angle formed by one side of a polygon and the line containing an adjacent side of the polygon. The angle is outside the polygon.



"Advanced" Definitions:

Simple Polygon: *a polygon whose sides do not intersect at any location other than its endpoints.* Simple polygons always divide a plane into two regions – one inside the polygon and one outside the polygon.

Complex Polygon: *a polygon with sides that intersect someplace other than their endpoints* (i.e., not a simple polygon). Complex polygons do not always have well-defined insides and outsides.

Skew Polygon: *a polygon for which not all of its vertices lie on the same plane.*

How Many Diagonals Does a Convex Polygon Have?

Believe it or not, this is a common question with a simple solution. Consider a polygon with n sides and, therefore, n vertices.

- Each of the n vertices of the polygon can be connected to $(n-3)$ other vertices with diagonals. That is, it can be connected to all other vertices except itself and the two to which it is connected by sides. So, there are $[\,n \cdot (n-3)]$ lines to be drawn as diagonals.
- However, when we do this, we draw each diagonal twice because we draw it once from each of its two endpoints. So, the number of diagonals is actually half of the number we calculated above.
- Therefore, **the number of diagonals in an n-sided polygon is:**

$$\frac{n \cdot (n-3)}{2}$$

Pre-Algebra
Interior and Exterior Angles of a Polygon

Interior Angles

The sum of the interior angles in an n-sided polygon is:

$$\sum = (n-2) \cdot 180^\circ$$

If the polygon is regular, you can calculate the measure of each interior angle as:

$$\frac{(n-2) \cdot 180^\circ}{n}$$

Notation: The Greek letter **"Σ"** is equivalent to the English letter **"S"** and is math short-hand for a summation (i.e., addition) of things.

Interior Angles		
Sides	Sum of Interior Angles	Each Interior Angle
3	180^0	60^0
4	360^0	90^0
5	540^0	108^0
6	720^0	120^0
7	900^0	129^0
8	$1{,}080^0$	135^0
9	$1{,}260^0$	140^0
10	$1{,}440^0$	144^0

Exterior Angles

No matter how many sides there are in a polygon, the sum of the exterior angles is:

$$\sum = 360^0$$

If the polygon is regular, you can calculate the measure of each exterior angle as:

$$\frac{360^0}{n}$$

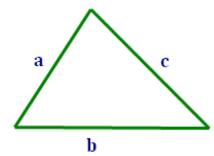
Exterior Angles		
Sides	Sum of Exterior Angles	Each Exterior Angle
3	360^0	120^0
4	360^0	90^0
5	360^0	72^0
6	360^0	60^0
7	360^0	51^0
8	360^0	45^0
9	360^0	40^0
10	360^0	36^0

Pre-Algebra
Perimeter and Area of a Triangle

Perimeter of a Triangle

The perimeter of a triangle is simply the sum of the measures of the three sides of the triangle.

$$P = a + b + c$$

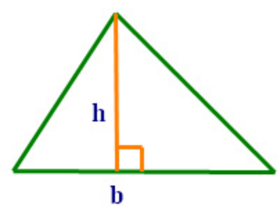


Area of a Triangle

There are two formulas for the area of a triangle, depending on what information about the triangle is available.

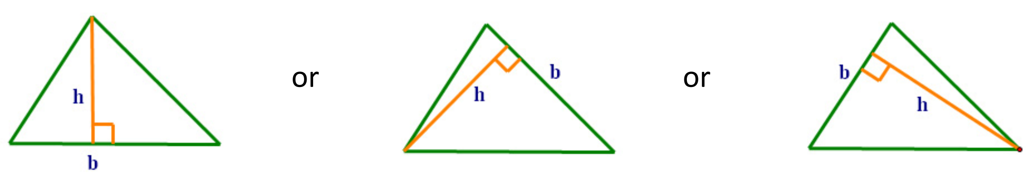
Formula 1: The formula most familiar to the student can be used when the base and height of the triangle are either known or can be determined.

$$A = \frac{1}{2}bh$$

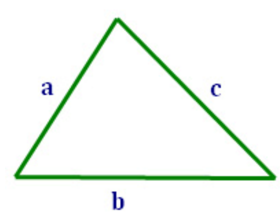


where, b is the length of the base of the triangle.
h is the height of the triangle.

Note: The base can be any side of the triangle. The height is the measure of the altitude of whichever side is selected as the base. So, you can use:



Formula 2: Another formula for the area of a triangle can be used when the lengths of all of the sides are known. Sometimes this formula, though less appealing, can be very useful.

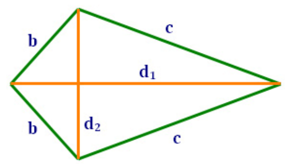
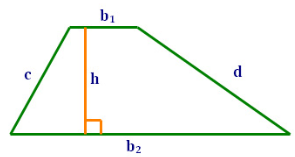
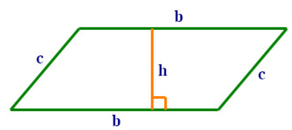
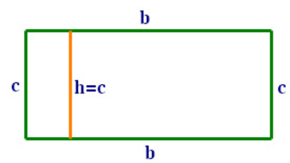
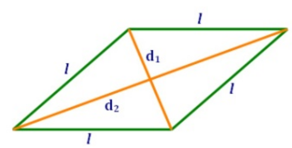
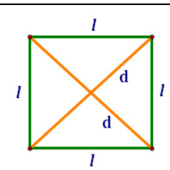


$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where, $s = \frac{1}{2}P = \frac{1}{2}(a + b + c)$. **Note:** s is sometimes called the semi-perimeter of the triangle

a, b, c are the lengths of the sides of the triangle.

Pre-Algebra
Perimeter and Area of Quadrilaterals

Name	Illustration	Perimeter	Area
Kite		$P = 2a + 2b$	$A = \frac{1}{2}(d_1 d_2)$
Trapezoid		$P = b_1 + b_2 + a + c$	$A = \frac{1}{2}(b_1 + b_2)h$
Parallelogram		$P = 2a + 2b$	$A = bh$
Rectangle		$P = 2a + 2b$	$A = bh$
Rhombus		$P = 4s$	$A = bh = \frac{1}{2}(d_1 d_2)$
Square		$P = 4s$	$A = s^2 = \frac{1}{2}(d^2)$

Pre-Algebra
Circle Lengths and Areas

Circumference and Area

$\boldsymbol{C = 2\pi \cdot r}$ is the **circumference** (i.e., the perimeter) of the circle.

$\boldsymbol{A = \pi r^2}$ is the **area** of the circle.

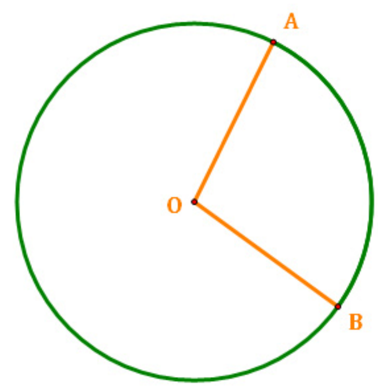
where: r is the radius of the circle.

Length of an Arc on a Circle

A common problem in the geometry of circles is to measure the length of an arc on a circle.

Definition: An arc is a segment along the circumference of a circle.

$$\boldsymbol{arc\ length = \frac{m\widehat{AB}}{360} \cdot C}$$



where: $m\angle\widehat{AB}$ is the measure (in degrees) of the arc. Note that this is also the measure of the central angle $\angle AOB$.

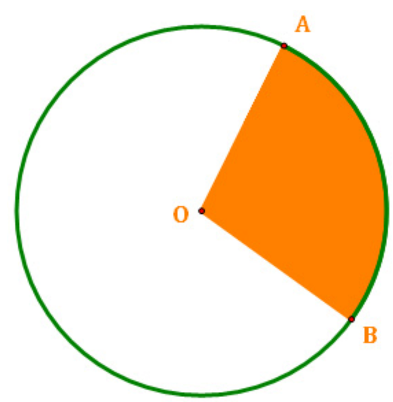
C is the circumference of the circle.

Area of a Sector of a Circle

Another common problem in the geometry of circles is to measure the area of a sector a circle.

Definition: A sector is a region in a circle that is bounded by two radii and an arc of the circle.

$$\boldsymbol{sector\ area = \frac{m\widehat{AB}}{360} \cdot A}$$

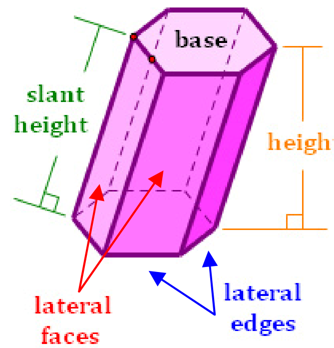


where: $m\angle\widehat{AB}$ is the measure (in degrees) of the arc. Note that this is also the measure of the central angle $\angle AOB$.

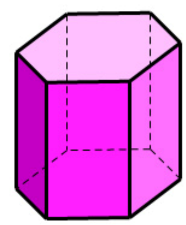
A is the area of the circle.

Pre-Algebra
Prisms

Definitions



- A **Prism** is a polyhedron with two congruent polygonal faces that lie in parallel planes.
- The **Bases** are the parallel polygonal faces.
- The **Lateral Faces** are the faces that are not bases.
- The **Lateral Edges** are the edges between the lateral faces.
- The **Slant Height** is the length of a lateral edge. Note that all lateral edges are the same length.
- The **Height** is the perpendicular length between the bases.



Right Hexagonal Prism

- A **Right Prism** is one in which the angles between the bases and the lateral edges are right angles. Note that in a right prism, the height and the slant height are the same.
- An **Oblique Prism** is one that is not a right prism.

- The **Surface Area** of a prism is the sum of the areas of all its faces.
- The **Lateral Area** of a prism is the sum of the areas of its lateral faces.

Surface Area and Volume of a Right Prism

Surface Area: $SA = Ph + 2B$

Lateral SA: $SA = Ph$

Volume: $V = Bh$

where, $P = \textit{the perimeter of the base}$
$h = \textit{the height of the prism}$
$B = \textit{the area of the base}$

Cavalieri's Principle

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume. This principle allows us to derive a formula for the volume of an oblique prism from the formula for the volume of a right prism.

Surface Area and Volume of an Oblique Prism

Surface Area: $SA = Ps + 2B$

Lateral SA: $SA = Ps$

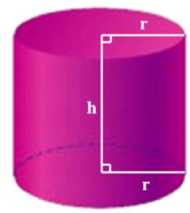
Volume: $V = Bh$

where, $P = \textit{the perimeter of the base}$
$s = \textit{the slant height of the prism}$
$h = \textit{the height of the prism}$
$B = \textit{the area of the base}$

Pre-Algebra
Cylinders

Definitions

- A **Cylinder** is a figure with two congruent circular bases in parallel planes.
- A cylinder has only one **Lateral Surface**. When deconstructed, the lateral surface of a cylinder is a rectangle with length equal to the circumference of the base.
- There are no **Lateral Edges** in a cylinder.
- The **Slant Height** is the length of the lateral side between the bases. Note that all lateral distances are the same length. The slant height has applicability only if the cylinder is oblique.
- The **Height** is the perpendicular length between the bases.



- A **Right Cylinder** is one in which the angles between the bases and the lateral side are right angles. Note that in a right cylinder, the height and the slant height are the same.
- An **Oblique Cylinder** is one that is not a right cylinder.

- The **Surface Area** of a cylinder is the sum of the areas of its bases and its lateral surface.
- The **Lateral Area** of a cylinder is the areas of its lateral surface.

Surface Area and Volume of a Right Cylinder

Surface Area: $\boldsymbol{SA = Ch + 2B}$

$\boldsymbol{= 2\pi rh + 2\pi r^2}$

Lateral SA: $\boldsymbol{SA = Ch = 2\pi rh}$

Volume: $\boldsymbol{V = Bh = \pi r^2 h}$

where, $C = the\ circumference\ of\ the\ base$
$h = the\ height\ of\ the\ cylinder$
$B = the\ area\ of\ the\ base$
$r = the\ radius\ of\ the\ base$

Surface Area and Volume of an Oblique Prism

Surface Area: $\boldsymbol{SA = Cs + 2B}$

$\boldsymbol{= 2\pi rs + 2\pi r^2}$

Lateral SA: $\boldsymbol{SA = Cs = 2\pi rs}$

Volume: $\boldsymbol{V = Bh = \pi r^2 h}$

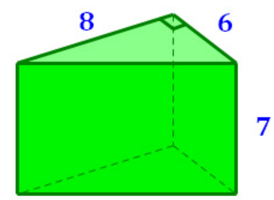
where, $C = the\ circumference\ of\ the\ base$
$s = the\ slant\ height\ of\ the\ cylinder$
$h = the\ height\ of\ the\ cylinder$
$B = the\ area\ of\ the\ base$
$r = the\ radius\ of\ the\ base$

Pre-Algebra
Surface Area by Decomposition

Sometimes the student is asked to calculate the surface are of a prism that does not quite fit into one of the categories for which an easy formula exists. In this case, the answer may be to decompose the prism into its component shapes, and then calculate the areas of the components. Note: this process also works with cylinders and pyramids.

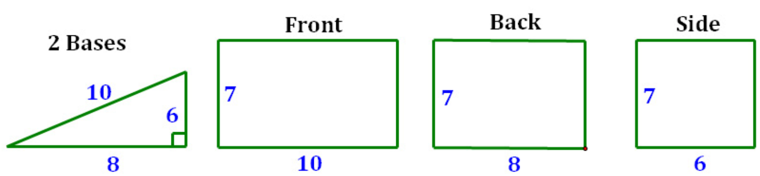
Decomposition of a Prism

To calculate the surface area of a prism, decompose it and look at each of the prism's faces individually.



Example: Calculate the surface area of the triangular prism at right.

To do this, first notice that we need the value of the hypotenuse of the base. Use the Pythagorean Theorem or Pythagorean Triples to determine the missing value is $\boldsymbol{10}$. Then, decompose the figure into its various faces:



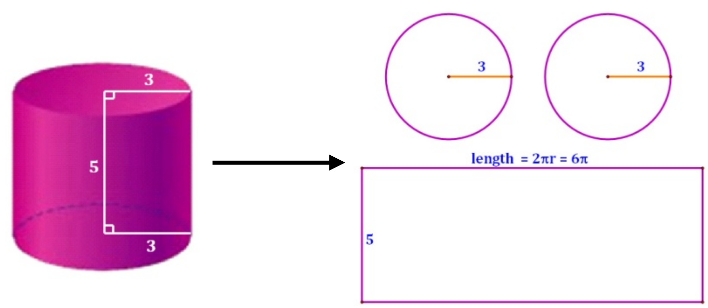
The surface area, then, is calculated as:

$$SA = (2\ Bases) + (Front) + (Back) + (Side)$$

$$SA = 2 \cdot \left(\frac{1}{2} \cdot 6 \cdot 8\right) + (10 \cdot 7) + (8 \cdot 7) + (6 \cdot 7) = 216$$

Decomposition of a Cone

The cylinder at right is decomposed into two circles (the bases) and a rectangle (the lateral face).



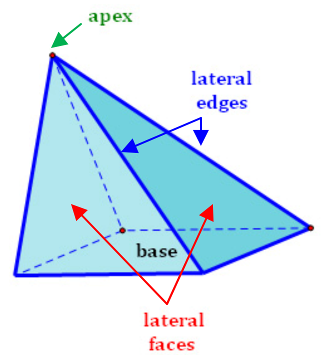
The surface area, then, is calculated as:

$$SA = (2\ tops) + (lateral\ face)$$

$$SA = 2 \cdot (\pi \cdot 3^2) + (6\pi \cdot 5) = 48\pi \sim 150.80$$

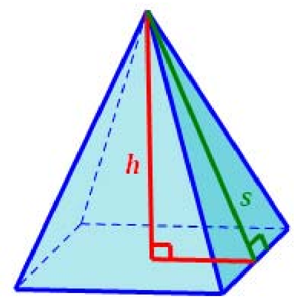
Pre-Algebra
Pyramids

Pyramids



- A **Pyramid** is a polyhedron in which the base is a polygon and the lateral sides are triangles with a common vertex.
- The **Base** is a polygon of any size or shape.
- The **Lateral Faces** are the faces that are not the base.
- The **Lateral Edges** are the edges between the lateral faces.
- The **Apex** of the pyramid is the intersection of the lateral edges. It is the point at the top of the pyramid.
- The **Slant Height** of a regular pyramid is the altitude of one of the lateral faces.
- The **Height** is the perpendicular length between the base and the apex.

- A **Regular Pyramid** is one in which the lateral faces are congruent triangles. The height of a regular pyramid intersects the base at its center.
- An **Oblique Pyramid** is one that is not a right pyramid. That is, the apex is not aligned directly above the center of the base.



- The **Surface Area** of a pyramid is the sum of the areas of all its faces.
- The **Lateral Area** of a pyramid is the sum of the areas of its lateral faces.

Surface Area and Volume of a Regular Pyramid

Surface Area: $SA = \frac{1}{2}Ps + B$

Lateral SA: $SA = \frac{1}{2}Ps$

Volume: $V = \frac{1}{3}Bh$

where, $P = the\ perimeter\ of\ the\ base$
$s = the\ slant\ height\ of\ the\ pyramid$
$h = the\ height\ of\ the\ pyramid$
$B = the\ area\ of\ the\ base$

Surface Area and Volume of an Oblique Pyramid

Surface Area: $SA = LSA + B$

Volume: $V = \frac{1}{3}Bh$

where, $LSA = the\ lateral\ surface\ area$
$h = the\ height\ of\ the\ pyramid$
$B = the\ area\ of\ the\ base$

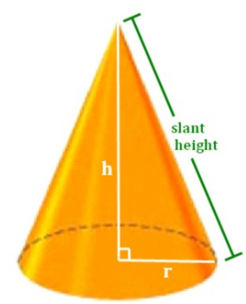
The lateral surface area of an oblique pyramid is the sum of the areas of the faces, which must be calculated individually.

Pre-Algebra
Cones

Definitions

- A **Circular Cone** is a 3-dimensional geometric figure with a circular base which tapers smoothly to a vertex (or apex). The apex and base are in different planes. Note: there is also an elliptical cone that has an ellipse as a base, but that will not be considered here.
- The **Base** is a circle.
- The **Lateral Surface** is area of the figure between the base and the apex.
- There are no **Lateral Edges** in a cone.
- The **Apex** of the cone is the point at the top of the cone.
- The **Slant Height** of a cone is the length along the lateral surface from the apex to the base.
- The **Height** is the perpendicular length between the base and the apex.

- A **Right Cone** is one in which the height of the cone intersects the base at its center.
- An **Oblique Cone** is one that is not a right cone. That is, the apex is not aligned directly above the center of the base.



- The **Surface Area** of a cone is the sum of the area of its lateral surface and its base.
- The **Lateral Area** of a cone is the area of its lateral surface.

Surface Area and Volume of a Right Cone

Surface Area: $SA = \pi rs + \pi r^2$

Lateral SA: $SA = \pi rs$

Volume: $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$

where, $r = the\ radius\ of\ the\ base$
$s = the\ slant\ height\ of\ the\ cone$
$h = the\ height\ of\ the\ cone$
$B = the\ area\ of\ the\ base$

Surface Area and Volume of an Oblique Cone

Surface Area: $SA = LSA + \pi r^2$

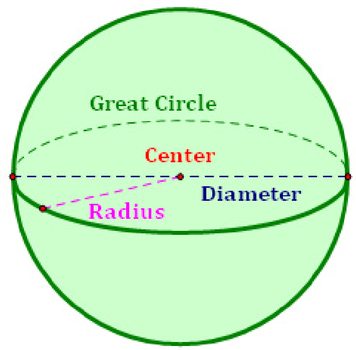
Volume: $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$

where, $LSA = the\ lateral\ surface\ area$
$r = the\ radius\ of\ the\ base$
$h = the\ height\ of\ the\ cone$

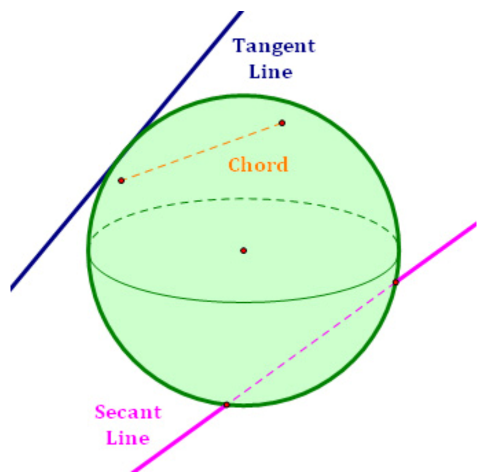
There is no easy formula for the lateral surface area of an oblique cone.

Pre-Algebra
Spheres

Definitions



- A **Sphere** is a 3-dimensional geometric figure in which all points are a fixed distance from a point. A good example of a sphere is a ball.
- **Center** – the middle of the sphere. All points on the sphere are the same distance from the center.
- **Radius** – a line segment with one endpoint at the center and the other endpoint on the sphere. The term "radius" is also used to refer to the distance from the center to the points on the sphere.
- **Diameter** – a line segment with endpoints on the sphere that passes through the center.



- **Great Circle** – the intersection of a plane and a sphere that passes through the center.
- **Hemisphere** – half of a sphere. A great circle separates a plane into two hemispheres.
- **Secant Line** – a line that intersects the sphere in exactly one point.
- **Tangent Line**– a line that intersects the sphere in exactly two points.
- **Chord** – a line segment with endpoints on the sphere that does not pass through the center.

Surface Area and Volume of a Sphere

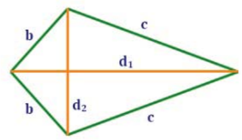
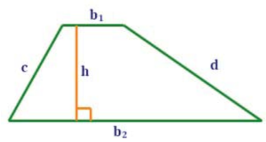
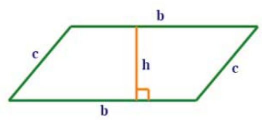
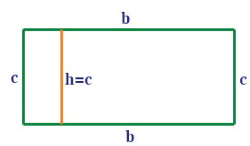
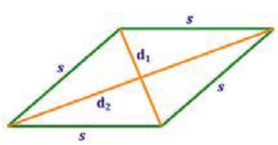
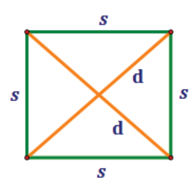
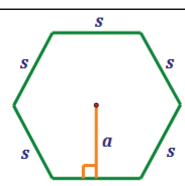
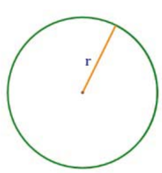
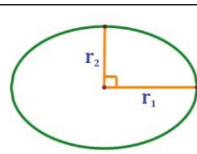
Surface Area: $\boldsymbol{SA = 4\pi r^2}$

Volume: $\boldsymbol{V = \frac{4}{3}\pi r^3}$

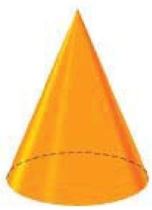
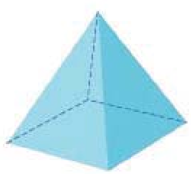
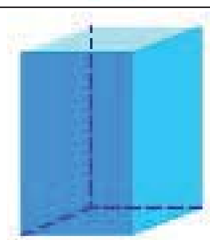
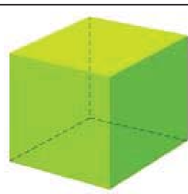
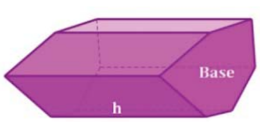
where, $r = the\ radius\ of\ the\ sphere$

Geometry

Summary of Perimeter and Area Formulas – 2D Shapes

Shape	Figure	Perimeter	Area
Kite		$P = 2b + 2c$ $b, c = sides$	$A = \frac{1}{2}(d_1 d_2)$ $d_1, d_2 = diagonals$
Trapezoid		$P = b_1 + b_2 + c + d$ $b_1, b_2 = bases$ $c, d = sides$	$A = \frac{1}{2}(\mathrm{b}_1 + \mathrm{b}_2)\mathrm{h}$ $\mathrm{b}_1, \mathrm{b}_2 = \text{bases}$ $\mathrm{h} = \text{height}$
Parallelogram		$P = 2b + 2c$ $b, c = sides$	$\mathrm{A} = \mathrm{bh}$ $b = base$ $h = height$
Rectangle		$P = 2b + 2c$ $b, c = sides$	$\mathrm{A} = \mathrm{bh}$ $b = base$ $h = height$
Rhombus		$P = 4s$ $s = side$	$A = bh = \frac{1}{2}(d_1 d_2)$ $d_1, d_2 = diagonals$
Square		$P = 4s$ $s = side$	$A = s^2 = \frac{1}{2}(d_1 d_2)$ $d_1, d_2 = diagonals$
Regular Polygon		$P = ns$ $n = number\ of\ sides$ $s = side$	$A = \frac{1}{2}\ a \cdot P$ $a = apothem$ $P = perimeter$
Circle		$C = 2\pi r = \pi d$ $r = radius$ $d = diameter$	$A = \pi r^2$ $r = radius$
Ellipse		$P \approx 2\pi\sqrt{\frac{1}{2}({r_1}^2 + {r_2}^2)}$ $r_1 = major\ axis\ radius$ $r_2 = minor\ axis\ radius$	$A = \pi r_1 r_2$ $r_1 = major\ axis\ radius$ $r_2 = minor\ axis\ radius$

Geometry
Summary of Surface Area and Volume Formulas – 3D Shapes

Shape	Figure	Surface Area	Volume
Sphere		$SA = 4\pi r^2$ $r = radius$	$V = \frac{4}{3}\pi r^3$ $r = radius$
Right Cylinder		$SA = 2\pi rh + 2\pi r^2$ $h = height$ $r = radius\ of\ base$	$V = \pi r^2 h$ $h = height$ $r = radius\ of\ base$
Cone		$SA = \pi rl + \pi r^2$ $l = slant\ height$ $r = radius\ of\ base$	$V = \frac{1}{3}\pi r^2 h$ $h = height$ $r = radius\ of\ base$
Square Pyramid		$SA = 2sl + s^2$ $s = base\ side\ length$ $l = slant\ height$	$V = \frac{1}{3}s^2 h$ $s = base\ side\ length$ $h = height$
Rectangular Prism		$SA = 2 \cdot (lw + lh + wh)$ $l = length$ $w = width$ $h = height$	$V = lwh$ $l = length$ $w = width$ $h = height$
Cube		$SA = 6s^2$ $s = side\ length\ (all\ sides)$	$V = s^3$ $s = side\ length\ (all\ sides)$
General Right Prism		$SA = Ph + 2B$ $P = Perimeter\ of\ Base$ $h = height\ (or\ length)$ $B = area\ of\ Base$	$V = Bh$ $B = area\ of\ Base$ $h = height$

Addition Table

0	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

Multiplication Table

0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

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