

## CASTING OUT NINES: THE BEST CHECK

The **basic rule** for casting out nines is to **add all the digits of a number**. If that sum exceeds nine, then add the digits of the sum you obtained. Continue until a single digit is obtained. If that single digit is **nine** then replace it with **zero**. The end result is a single digit in the range zero to eight.

Example: 459872164 -> 46 -> 10 -> 1

Then 1 is the cast out digit (COD for short). This may seem like lots of work and **indeed it is!** **BUT** there are **several** ways of speeding this up immensely.

### TIME SAVING TIP NUMBER 1:

**Before** summing the digits of the number **cross out** ("cast out") any two (or more) digits that add to 9, 18, 27, etc. Then add the remaining digits.

Examples: The example above, 459872164, has several nine's combinations that can be cast out: 4&5, 9, 8&1, 7&2. This only leaves the 6 and 4 to be added. We get 10 and then 1. It is common to **lightly** cross out the nines combinations that we are casting out: ~~459872164~~ -> 10 -> 1. (It is easier to lightly cross out by hand than on a wordprocessor.)

2&3&4, 6&6&6, 5&6&7, 2&2&5, 2&8&8 are some **triple** combinations we can cast out. Some of these add to nine whereas others add to eighteen (two nines).

~~6466~~ -> 4      ~~28384~~ -> 88 -> 16 -> 7      ~~2858~~ -> 5

### TIME SAVING TIP NUMBER 2:

After casting out all combinations you can then begin adding the digits that remain. **Anytime** (even though you have not yet added **all** the digits) that partial sum exceeds nine, add the digits of the partial sum.

7853      7+8=15 -> 6    Then 6+5=11 -> 2    Then 2+3=5

Of course we could have cast out the 7&8&3 since this adds to 18 thus leaving the 5 immediately.

### TIME SAVING TIP NUMBER 3:

**Snitch** from a small digit to add to a larger digit to make a nines combination. Add in what is left of the smaller digit.

Examples:

- 743** Snitch 2 from the 4 and add it to the 7 thus obtaining 9&2. Then cast out the nine and add the remaining 2 (from the 4) to the 3 to get the COD of 5.
- 875** Snitch 1 and 2 from the 5 to get 8&1 and 7&2. We then have left 2 of the 5. So 2 is the COD.
- 678** Snitch 1 from the 8 and add it to the 6 thus obtaining three sevens. Three sevens is 21-->3 by adding 2&1.

Any time a partial sum exceeds nine, we can subtract nine from that sum. This will give the same result as adding the digits of the partial sum. Personally I prefer adding the digits of the partial sum.

With a bit of practice you can **tailor** the process to your own liking and obtain COD's rapidly and accurately. Then these digits can be used to **check** addition, subtraction, division, multiplication and factoring.

The great **advantage** to this technique is that the check only involves **single** digits whereas the original problem usually involves **multiple** digit numbers. In all of these except division we do the **same operation** on the **cast out digits** as we did on the original problem.

### USING CO9's TO CHECK A PROBLEM:

1. For each number in the original problem obtain a single digit by casting out nines.
2. For +, - and \* perform on these digits the same operation(s) you performed on the **original** numbers and cast out nines to obtain a **single digit**.. (For division see example below.)
3. By casting out nines obtain a single digit from the answer to the **original** problem.

**If** the digits obtained in steps 2 and 3 are the **same**, then the problem **checks**. If they are **different**, then something is **wrong**. The problem or the check (or both) is wrong.

<u>Multiply:</u>	<u>Add:</u>	<u>Subtract:</u>	<u>Subtract:</u>	
124 7	2873 11 2	7361 8	2433 3+9=12	Add
*211 *4	+5738 +5	-2534 -5	-1726 -7 -7	9 if
26164 28	8611 7	4827 3	707 5	top is
10 10	7 --> 7	12	14	smaller
1 -> 1		3 --> 3	5 -----> 5	

For subtraction if the **lower digit** is larger than the **upper digit**, then **add nine** to the upper digit **before** subtracting. If we can cast out nines with impunity, then we can certainly add them back in.

**Divide:**

	257		?	
(a little bit different)	24	6182	$24 \times 257 + 14$	$= 6182$ (usual check)
		48	$6 \times 5 + 5$	$\text{----> } 8$ (CO9's check)
		138	$30 + 5$	
		120	$35$	
		182	$8$	$\text{-----> } 8$
		168		
		14		

Note that for division the CO9's check follows the usual check except we get to use the single digits obtained from the four numbers in the usual check.

Using CO9's to check these arithmetic problems is highly accurate **when not more than one** mistake is made in the calculations. If two or more mistakes are made then the CO9's check is about 89% effective since there is a 1 in 9 chance of the COD's matching by coincidence. **No checking method** is 100% effective since the person doing the checking is subject to making mistakes in calculations or in entering numbers into devices.

In my experience CO9's is the **fastest check** and is also as accurate or more so than most other methods of checking.

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Also there is a really **great bonus** (something that most math teachers are not aware of) to learning how to CO9's in arithmetic. We can check addition, subtraction, multiplication, division and factoring of **polynomials** in **algebra** by CO9's. The reason? The coefficients of the terms in the polynomials are written in **base ten**.

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So we do just like we do in arithmetic except we usually have **negative** coefficients as well as **positive** coefficients. To cast out nines on a polynomial the basic method is to add all the coefficients and CO9's on that sum. But of course we can also use time saving tips on them as well. It is actually **a bit easier** for the polynomials since the positive and negative digits **sometimes cancel** out giving a **zero** instead of 9, 18, 27, etc. We will explain more about this in a future short article.

Note that if the sum is negative, for example -26, then the cast out digit is  $-(2+6)=-8$ . We can then add 9 to this to obtain a positive cast out digit of 1 if we desire.

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CO9sHW1.wpd

**Cast Out Nines.**

97898 -> 74648 -> 83805 -> 62248 -> 87619 ->  
12044 -> 31177 -> 77016 -> 18735 -> 79643 ->  
26451 -> 66395 -> 85851 -> 88312 -> 28579 ->  
8905577 -> 7968540 -> 4881467 -> 3253238 ->  
1710572 -> 4870441 -> 1759973 -> 3612057 ->  
8557903 -> 1182012 -> 4721794 -> 4767719 ->  
388877354 -> 477645100 -> 395144211 ->  
911432466 -> 939415685 -> 145107320 ->  
513876538 -> 335737523 -> 941048719 ->

**Cast Out Nines:**

72784 -> 38583 -> 56341 -> 72843 -> 78691 ->  
62774 -> 26276 -> 36347 -> 57883 -> 18080 ->  
97532 -> 25263 -> 86640 -> 38992 -> 29916 ->  
7121368 -> 2459222 -> 5556578 -> 7252766 ->  
1049758 -> 1582699 -> 3858156 -> 5041241 ->  
3479678 -> 4401583 -> 7671879 -> 8640611 ->  
666236776 -> 890764642 -> 220490383 ->  
475178579 -> 924179169 -> 159323175 ->  
236289754 -> 930731019 -> 256669191 ->

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CO9sHW1.wpd

**Cast Out Nines. (Answer Key)**

97898 -> 5    74648 -> 2    83805 -> 6    62248 -> 4    87619 -> 4  
12044 -> 2    31177 -> 1    77016 -> 3    18735 -> 6    79643 -> 2  
26451 -> 0    66395 -> 2    85851 -> 0    88312 -> 4    28579 -> 4  
8905577 -> 5    7968540 -> 3    4881467 -> 2    3253238 -> 8  
1710572 -> 5    4870441 -> 1    1759973 -> 5    3612057 -> 6  
8557903 -> 1    1182012 -> 6    4721794 -> 7    4767719 -> 5  
388877354 -> 8    477645100 -> 7    395144211 -> 3  
911432466 -> 0    939415685 -> 5    145107320 -> 5  
513876538 -> 1    335737523 -> 2    941048719 -> 7

**Cast Out Nines:**

72784 -> 1    38583 -> 0    56341 -> 1    72843 -> 6    78691 -> 4  
62774 -> 8    26276 -> 5    36347 -> 5    57883 -> 4    18080 -> 8  
97532 -> 8    25263 -> 0    86640 -> 6    38992 -> 4    29916 -> 0  
7121368 -> 1    2459222 -> 8    5556578 -> 5    7252766 -> 8  
1049758 -> 7    1582699 -> 4    3858156 -> 0    5041241 -> 8  
3479678 -> 8    4401583 -> 7    7671879 -> 0    8640611 -> 8  
666236776 -> 4    890764642 -> 1    220490383 -> 4  
475178579 -> 8    924179169 -> 3    159323175 -> 0  
236289754 -> 1    930731019 -> 6    256669191 -> 0

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**Add** and check by casting out nines.

635	879	720	6109	5829
<u>427</u>	<u>288</u>	<u>153</u>	<u>3541</u>	<u>4961</u>

314	676	881	5960	9235
<u>131</u>	<u>608</u>	<u>501</u>	<u>1151</u>	<u>7963</u>

**Subtract** and check by casting out nines.

921	849	777	9387	5504
<u>731</u>	<u>190</u>	<u>480</u>	<u>1200</u>	<u>1978</u>

962	844	900	9848	7867
<u>557</u>	<u>204</u>	<u>235</u>	<u>9060</u>	<u>4518</u>

**Divide** and check by casting out nines.

19 $\overline{)5140}$	41 $\overline{)4564}$	792 $\overline{)82675}$
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26 $\overline{)5490}$	15 $\overline{)4726}$	251 $\overline{)74338}$
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41 $\overline{)3368}$	82 $\overline{)9319}$	511 $\overline{)16552}$
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CO9sHW3.wpd

**Add** and check by casting out nines. (**Answer Key**)

(Should check with the **bold** digit given.)

We add the cast out digits and CO9's if greater than 8.

$$\begin{array}{r}
 635 \quad 5 \quad 879 \quad 6 \quad 720 \quad 0 \quad 6109 \quad 7 \quad 5829 \quad 6 \\
 \underline{427} \quad +4 \quad \underline{288} \quad +0 \quad \underline{153} \quad +0 \quad \underline{3541} \quad +4 \quad \underline{4961} \quad +2 \\
 1062 \quad 9 \quad 1167 \quad \mathbf{6} \quad 873 \quad \mathbf{0} \quad 9650 \quad 11 \quad 10790 \quad \mathbf{8} \\
 \mathbf{0} \qquad \qquad \qquad \mathbf{2}
 \end{array}$$

$$\begin{array}{r}
 314 \quad 8 \quad 676 \quad 1 \quad 881 \quad 8 \quad 5960 \quad 2 \quad 9235 \quad 1 \\
 \underline{131} \quad +5 \quad \underline{608} \quad +5 \quad \underline{501} \quad +6 \quad \underline{1151} \quad +8 \quad \underline{7963} \quad +7 \\
 13 \qquad \qquad \mathbf{6} \qquad \qquad 14 \qquad \qquad 10 \qquad \qquad \mathbf{8} \\
 \mathbf{4} \qquad \qquad \qquad \mathbf{5} \qquad \qquad \mathbf{1}
 \end{array}$$

**Subtract** and check by casting out nines.

(*Italics* indicates we had to add 9 before subtracting.)

$$\begin{array}{r}
 921 \quad 3 \quad 849 \quad 3 \quad 777 \quad 3 \quad 9387 \quad 9 \quad 5504 \quad 14 \\
 \underline{731} \quad -2 \quad \underline{190} \quad -1 \quad \underline{480} \quad -3 \quad \underline{1200} \quad -3 \quad \underline{1978} \quad -7 \\
 190 \quad \mathbf{1} \quad 659 \quad \mathbf{2} \quad 297 \quad \mathbf{0} \quad 8187 \quad \mathbf{6} \quad 3526 \quad \mathbf{7}
 \end{array}$$

$$\begin{array}{r}
 962 \quad 8 \quad 844 \quad 7 \quad 900 \quad 9 \quad 9848 \quad 11 \quad 7867 \quad 1 \\
 \underline{557} \quad -8 \quad \underline{204} \quad -6 \quad \underline{235} \quad -1 \quad \underline{9060} \quad -6 \quad \underline{4518} \quad -0 \\
 \mathbf{0} \qquad \qquad \mathbf{1} \qquad \qquad \mathbf{8} \qquad \qquad \mathbf{5} \qquad \qquad \mathbf{1}
 \end{array}$$

**Divide** and check by casting out nines.

CO9 on **divisor\*quotient+remainder** (= ? CO9 of dividend)

$$19 \overline{)5140} \qquad 41 \overline{)4564} \qquad 792 \overline{)82675}$$

$$1*0+1 \rightarrow \mathbf{1} \qquad 5*3+4 \rightarrow \mathbf{1} \qquad 0*5+1 \rightarrow \mathbf{1}$$

$$26 \overline{)5490} \qquad 15 \overline{)4726} \qquad 251 \overline{)74338}$$

$$8*4+4 \rightarrow \mathbf{0} \qquad 6*0+1 \rightarrow \mathbf{1} \qquad 8*8+6 \rightarrow \mathbf{7}$$

$$41 \overline{)3368} \qquad 82 \overline{)9319} \qquad 511 \overline{)16552}$$

$$5*1+6 \rightarrow \mathbf{2} \qquad 1*5+8 \rightarrow \mathbf{4} \qquad 7*5+2 \rightarrow \mathbf{1}$$

Calculations for last three problems:

$$41*82+6=?3368 \quad 82*113+53=?9319 \quad 511*32+200=?16552$$

CO9's on these to obtain:

$$5*1 + 6 \rightarrow 2 \qquad 1*5 + 8 \rightarrow 4 \qquad 7*5 + 2 \rightarrow 1$$

NAME: \_\_\_\_\_ DATE: \_\_\_\_\_ CO9Mult.wpd

**Multiply** and cast out nines to check:

53	654	2632
<u>73</u>	<u>342</u>	<u>9993</u>

12	963	4348
<u>92</u>	<u>656</u>	<u>368</u>

21	169	2941
<u>83</u>	<u>36</u>	<u>4946</u>

11	883	2954
<u>56</u>	<u>145</u>	<u>1332</u>

38	288	2333
<u>6</u>	<u>227</u>	<u>7236</u>

49	466	1271
<u>38</u>	<u>834</u>	<u>7545</u>

29	571	2526
<u>25</u>	<u>431</u>	<u>7389</u>



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CO9Mult.wpd

**Multiply** and cast out nines to check: **(Answer Key)**

$$\begin{array}{r} 53 \quad 8 \\ 73 \quad *1 \\ \hline 3869 \quad \mathbf{8} \end{array}$$

$$\begin{array}{r} 654 \quad 6 \\ 342 \quad *0 \\ \hline 223668 \quad \mathbf{0} \end{array}$$

$$\begin{array}{r} 2632 \quad 4 \\ 9993 \quad *3 \\ \hline 26301576 \quad 12 \\ \mathbf{3} \end{array}$$

$$\begin{array}{r} 12 \quad 3 \\ 92 \quad *2 \\ \hline \mathbf{6} \end{array}$$

$$\begin{array}{r} 963 \quad 0 \\ 656 \quad *8 \\ \hline \mathbf{0} \end{array}$$

$$\begin{array}{r} 4348 \quad 1 \\ 368 \quad *8 \\ \hline \mathbf{8} \end{array}$$

$$\begin{array}{r} 21 \quad 3 \\ 83 \quad *2 \\ \hline \mathbf{6} \end{array}$$

$$\begin{array}{r} 169 \quad 7 \\ 36 \quad *0 \\ \hline \mathbf{0} \end{array}$$

$$\begin{array}{r} 2941 \quad 7 \\ 4946 \quad *5 \\ \hline 35 \\ \mathbf{8} \end{array}$$

$$\begin{array}{r} 11 \quad 2 \\ 56 \quad *2 \\ \hline \mathbf{4} \end{array}$$

$$\begin{array}{r} 883 \quad 1 \\ 145 \quad *1 \\ \hline \mathbf{1} \end{array}$$

$$\begin{array}{r} 2954 \quad 2 \\ 1332 \quad *0 \\ \hline \mathbf{0} \end{array}$$

$$\begin{array}{r} 38 \quad 2 \\ 6 \quad *6 \\ \hline 12 \\ \mathbf{3} \end{array}$$

$$\begin{array}{r} 288 \quad 0 \\ 227 \quad *2 \\ \hline \mathbf{0} \end{array}$$

$$\begin{array}{r} 2333 \quad 2 \\ 7236 \quad *0 \\ \hline \mathbf{0} \end{array}$$

$$\begin{array}{r} 49 \quad 4 \\ 38 \quad *2 \\ \hline \mathbf{8} \end{array}$$

$$\begin{array}{r} 466 \quad 7 \\ 834 \quad *6 \\ \hline 42 \\ \mathbf{6} \end{array}$$

$$\begin{array}{r} 1271 \quad 2 \\ 7545 \quad *3 \\ \hline \mathbf{6} \end{array}$$

$$\begin{array}{r} 29 \quad 2 \\ 25 \quad *7 \\ \hline 14 \\ \mathbf{5} \end{array}$$

$$\begin{array}{r} 571 \quad 4 \\ 431 \quad *8 \\ \hline 32 \\ \mathbf{5} \end{array}$$

$$\begin{array}{r} 2526 \quad 6 \\ 7389 \quad *0 \\ \hline \mathbf{0} \end{array}$$