

GCD AND LCM FOR FRACTIONS

Greatest Common Divisors and **Least Common Multiplies** can be defined for fractions.

The **GCD** of $\frac{x}{y}$ and $\frac{u}{v}$ is defined as $\frac{\text{GCD}(x, u)}{\text{GCD}(y, v)}$.

$$\text{Example: GCD}\left(\frac{11}{25}, \frac{22}{35}\right) \text{ is } \frac{11}{5}$$

This can be used to find the **sum** (or difference) of two fractions **without** having to first obtain the least common denominator. For example

$$\frac{11}{25} + \frac{22}{35} = \frac{11}{5} \left(\frac{1}{5} + \frac{2}{7}\right) = \frac{11}{5} \left(\frac{7+10}{35}\right) = \frac{11}{5} * \frac{17}{35} = \frac{187}{175}$$

This could **really help** aspiring arithmetic and algebra students since it is easier to spot a **GCD** than an **LCM**. And note that this also works in algebra though getting a **LCD** could often be less writing.

$$\frac{12}{x^2y} - \frac{18}{xy^2} = \frac{6}{xy} \left(\frac{2}{x} - \frac{3}{y}\right) = \frac{6}{xy} \left(\frac{2y-3x}{xy}\right) = \frac{12y-18x}{x^2y^2}$$

The **LCM** of $\frac{x}{y}$ and $\frac{u}{v}$ is defined as $\frac{\text{LCM}(x, u)}{\text{LCM}(y, v)}$.

$$\text{Example: LCM}\left(\frac{11}{25}, \frac{22}{35}\right) \text{ is } \frac{22}{175}$$

And the usual **LCM(M, N) * GCD(M, N) = M * N** holds.