# POLYNOMIAL ARITHMETIC

**Polynomial Algebra** is in essence nothing more than **arithmetic in an unknown base**. Consider the example of expanded notation for 432, $432_5$ and $432_u$.

$$432 = 4x10^2 + 3x10 + 2$$

$$432_5 = 4x5^2 + 3x5 + 2$$

$$432_u = 4u^2 + 3u + 2$$

The expanded form of $432_u$ is what we call a **polynomial in the single variable u**. If we consider **negative digits** in place value in the usual base 10 arithmetic, then we have the analogue to polynomial algebra which is expanded notation for an unknown base arithmetic. For example

$$4\bar{3}2 = 4x10^2 + \bar{3}x10 + 2$$

$$= 4x10^2 - 3x10 + 2$$

$$4\bar{3}2_u = 4u^2 + \bar{3}u + 2$$

$$= 4u^2 - 3u + 2$$

Polynomial algebra then provides us with the **rules of arithmetic** that can be performed **without knowing what base** is being used. Polynomial algebra can thus be performed **in the short notation** in the same identical manner as arithmetic with one exception; namely, we must **omit** any operations that are base dependent.

For example, we must omit borrowing, carrying, enbarring and debarring. (Note: We must write the coefficients 4, -3 and 2 in SOME base, so we naturally choose base 10 for these coefficients. If the base were specified, then we could also express the coefficients in terms of this base.) Since there is no carrying in unknown base arithmetic it is possible to generate **multiple digit** coefficients. To handle these in short notation we simply **separate** the various coefficients by spaces. For example

$$4\ \bar{1}\bar{3}\ 7\ 12_u = 4u^3 + \bar{1}\bar{3}u^2 + 7u + 12$$

$$= 4u^3 - 13u^2 + 7u + 12$$

Missing powers of the base are indicated by **zeros** just like in base 10 arithmetic:

$$4\ 0\ 0\ \bar{3}\ 0\ 12 = 4u^5 - 3u^2 + 12$$

Since polynomial algebra in a single variable is the same thing as arithmetic in an unknown base, we shall call these **polyrith** for short. Because <u>there are **no base dependent** operations</u> in polyrith, the arithmetic is actually **easier** than base ten arithmetic.

Subtraction is done by adding the opposite (changing signs and adding) in algebra. If subtraction in base 10 arithmetic were also introduced as adding the opposite, then the student would already be familiar with this concept when he reached algebra. This method of subtraction together with cross multiplication makes addition, subtraction, multiplication and division of polynomials quite **easy** in the **short** notation.

**EXAMPLE: Given $2x^2 - 5x + 3$ and $x^2 - 4x - 4$**

**Addition**

$$\begin{array}{ccc} 2 & \bar{5} & 3 \\ \underline{1} & \underline{\bar{4}} & \underline{\bar{4}} \\ 3 & \bar{9} & \bar{1} \end{array}$$

$$3x^2 - 9x - 1$$

**Multiplication**

$$\begin{array}{ccc} 2 & \bar{5} & 3 \\ \underline{1} & \underline{\bar{4}} & \underline{\bar{4}} \\ \bar{8} & 20 & \bar{12} \end{array}$$

$$\begin{array}{ccccc} \bar{8} & 20 & \bar{12} & & \\ \hline 2 & \bar{5} & 3 & & \\ \hline 2 & \bar{13} & 15 & 8 & \bar{12} \end{array}$$

$$2x^4 - 13x^3 + 15x^2 + 8x - 12$$

**Subtraction**

$$\begin{array}{ccc} 2 & \bar{5} & 3 \\ \underline{1} & \underline{\bar{4}} & \underline{\bar{4}} \\ 1 & \bar{1} & 7 \end{array}$$

$$x^2 - x + 7$$

**EXAMPLE: Divide: $(4x^4 - 4x^3 - x^2 + x - 2) / (2x^2 + x - 1)$**

$$\begin{array}{ccc|ccccc} & & & & & 2 & \bar{3} & 2 \\ \hline 2 & 1 & \bar{1} & 4 & \bar{4} & \bar{1} & 1 & \bar{2} \\ & & & \underline{4} & \underline{2} & \underline{\bar{2}} & & \\ & & & & \bar{6} & 1 & 1 & \\ & & & & \underline{\bar{6}} & \underline{\bar{3}} & \underline{3} & \\ & & & & & 4 & \bar{2} & \bar{2} \\ & & & & & \underline{4} & \underline{2} & \underline{\bar{2}} \\ & & & & & & \bar{4} & 0 \end{array}$$

Quotient: $2\ \bar{3}\ 2 = 2x^2 - 3x + 2$

Remainder: $\bar{4}\ 0 = -4x$

Here are some exercises to help master the translations to and from the expanded (long) form of polynomials and the short form. Remember that terms that are missing in the long form must be replaced by a zero in the short form.

**Examples:** $3x^2-2 = 3\ 0\ \bar{2}$; $x^3+x = 1\ 0\ 1\ 0$; $\bar{2}x^4-3x-1 = \bar{2}\ 0\ 0\ \bar{3}\ \bar{1}$

1) $2\ \bar{3}$ = ______ $5\ 2$ = ______ $3\ 0$ = ______ $1\ 2$ = ______

2) $\bar{1}\ 4$ = ______ $0\ \bar{3}$ = ______ $\bar{1}\ \bar{1}$ = ______ $\bar{2}\ 7$ = ______

3) $\bar{2}\ 12$ = ______ $11\ \bar{2}$ = ______ $\bar{5}\ \overline{21}$ = ______ $14\ 2$ = ______

4) $3\ \bar{1}\ \bar{4}$ = __________ $\bar{5}\ 3\ 1$ = __________ $17\ 3\ \overline{20}$ = __________

5) $1\ 0\ \bar{3}$ = __________ $\bar{9}\ \bar{2}\ 0$ = __________ $11\ \overline{12}\ \bar{1}$ = __________

6) $\bar{4}\ 0\ 0$ = __________ $6\ \bar{1}\ 0$ = __________ $0\ \overline{123}\ 0$ = __________

7) $\bar{2}\ 0\ 0\ 3\ \bar{1}$ = _______________ $1\ \bar{3}\ 0\ \bar{4}\ 0$ = _______________

8) $0\ \bar{3}\ 2\ 0$ = _______________ $5\ 0\ 0\ 0\ \bar{1}$ = _______________

9) $1\ 1\ \bar{1}\ \bar{1}\ 1$ = _______________ $\bar{2}\ 3\ \bar{4}\ 5\ 6$ = _______________

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1) $x+2$ = ______ $3x$ = ______ $5x+2$ = ______ $2x-3$ = ______

2) $-2x+7$ = ______ $-x-1$ = ______ $-3$ = ______ $-x+4$ = ______

3) $14x+2$= ______ $-5x-21$= ______ $11x-2$= ______ $-2x+12$= ______

4) $17x^2+3x-20$=________ $-5x^2+3x+1$=________ $3x^2-x-4$=________

5) $11x^2-12x-1$=________ $-9x^2-2x$=________ $x^2-3$ = ________

6) $-123x$ = __________ $6x^2-x$ = __________ $-4x^2$ = __________

7) $x^4-3x^3-4x$ = _______________ $-2x^4+3x-1$ = _______________

8) $5x^4-1$ = _______________ $-3x^2+2x$ = _______________

9) $-2x^4+3x^3-4x^2+5x+6$=_______________ $x^4+x^3-x^2-x+1$ = __________

If you would like to see the answers to these exercises, they are the same problems top/bottom, but each line is in reversed order.